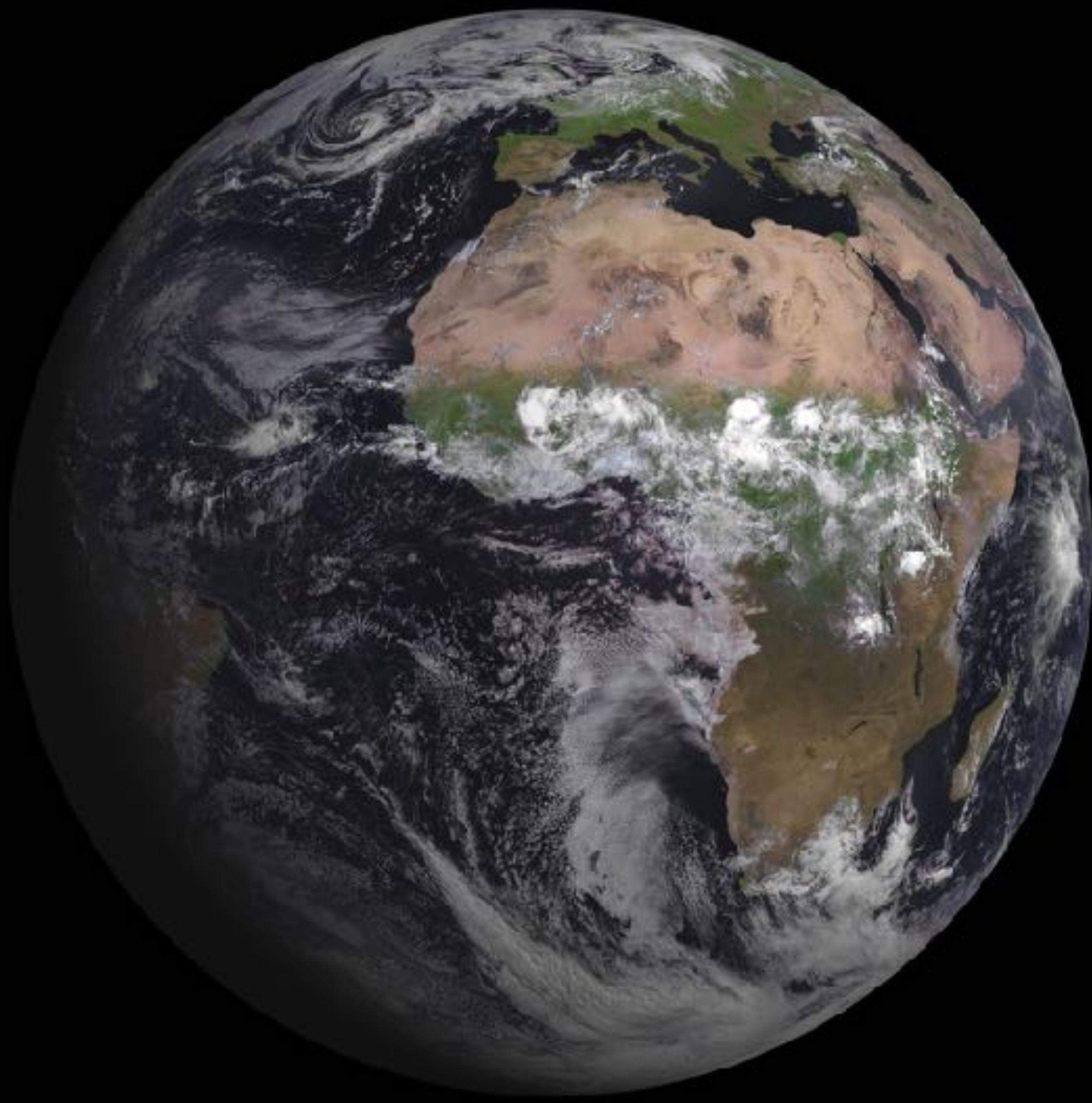
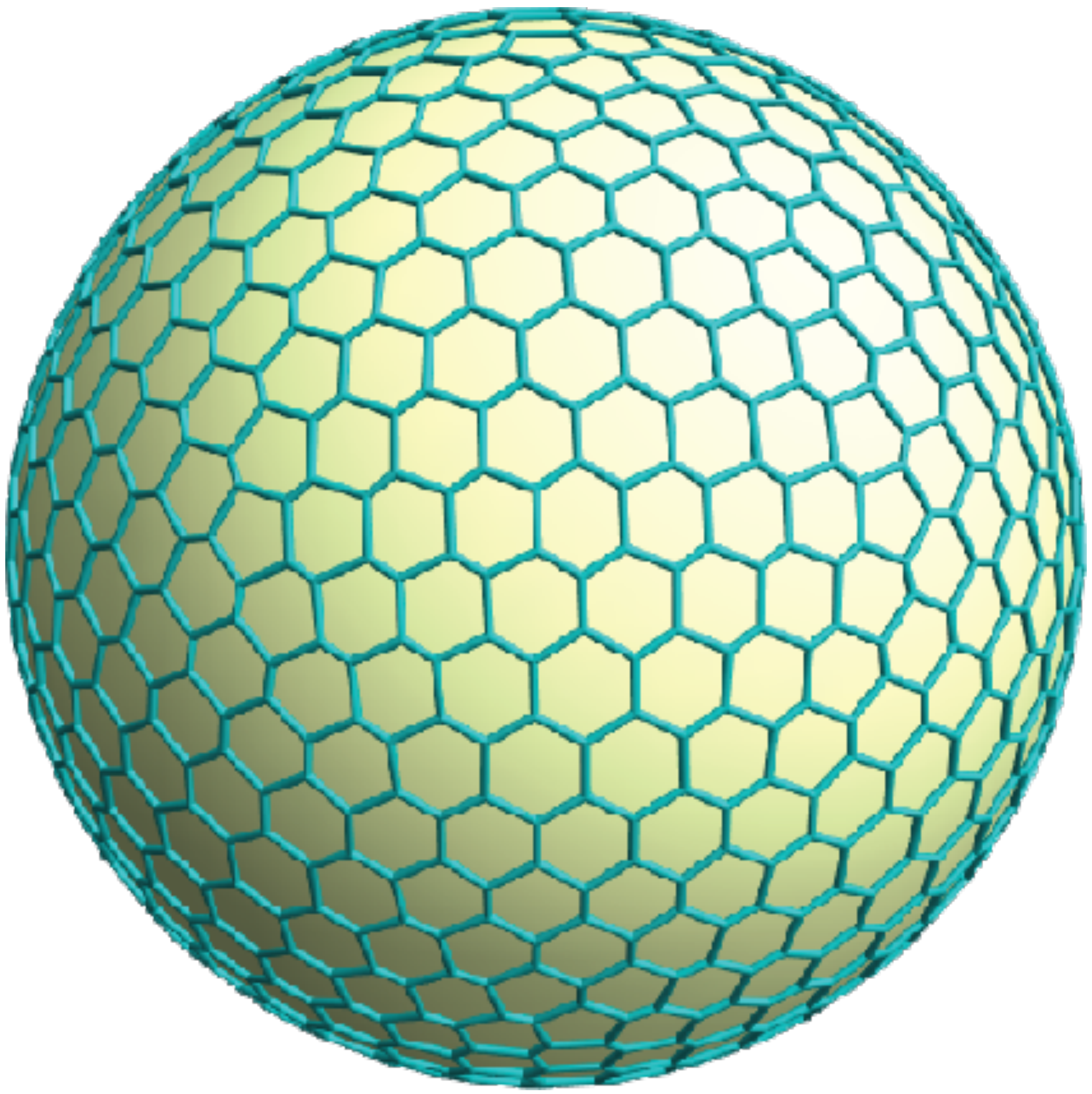
A Global Vector Vorticity Model On a Geodesic Grid



David Randall, Ross Heikes, Celal Konor

Geodesic Grid



Grid	No. of grid points N	Avg grid distance ℓ (km)
G0	12	6699.1
G1	42	3709.8
G2	162	1908.8
G3	642	961.4
G4	2562	481.6
G5	10 242	240.9
G6	40 962	120.4
G7	163 842	60.2
G8	655 362	30.1
G9	2 621 442	15.0
G10	10 485 762	7.53
G11	41 943 042	3.76
G12	167 772 162	1.88
G13	671 088 642	0.94

Non-hydrostatic regime

I will describe a new non-hydrostatic dynamical core intended for use as a global cloud-resolving model. The model uses a geodesic (hexagonal-pentagonal) grid.

The model uses the Unified System™ of Arakawa & Konor.

Fully compressible system	*Unified System*
$p \equiv p_{qs} + \delta p \quad \rho \equiv \rho_{qs} + \delta\rho$ $\pi \equiv \pi_{qs} + \delta\pi \quad T \equiv T_{qs} + \delta T$	$p \equiv p_{qs} + \delta p \quad \rho \equiv \rho_{qs} + \delta\rho$ $\pi \equiv \pi_{qs} + \delta\pi \quad T \equiv T_{qs} + \delta T$
Horizontal momentum equation: $\frac{D\mathbf{v}}{Dt} + f\mathbf{k}\times\mathbf{v} = -c_p\theta\nabla_H\left(\pi_{qs} + \delta\pi\right) + \mathbf{F_v}$	**Horizontal momentum equation:** $\frac{D\mathbf{v}}{Dt} + f\mathbf{k}\times\mathbf{v} = -c_p\theta\nabla_H\left(\pi_{qs} + \delta\pi\right) + \mathbf{F_v}$
Vertical momentum equation: $\frac{Dw}{Dt} = -c_p\theta\frac{\partial\delta\pi}{\partial z} + F_w$	**Vertical momentum equation:** $\frac{Dw}{Dt} = -c_p\theta\frac{\partial\delta\pi}{\partial z} + F_w$
Thermodynamic equation: $\frac{D\theta}{Dt} = \frac{Q}{c_p\pi}$	**Thermodynamic equation:** $\frac{D\theta}{Dt} = \frac{Q}{c_p\pi}$
Continuity equation: $\frac{\partial\left(\rho_{qs} + \delta\rho\right)}{\partial t} = -\nabla_H\cdot\left[\left(\rho_{qs} + \delta\rho\right)\mathbf{v}\right] - \frac{\partial\left[\left(\rho_{qs} + \delta\rho\right)w\right]}{\partial z}$	**Continuity equation:** $\frac{\partial\rho_{qs}}{\partial t} = -\nabla_H\cdot\left(\rho_{qs}\mathbf{v}\right) - \frac{\partial\left(\rho_{qs}w\right)}{\partial z}$ $\rho_{qs} \gg \delta\rho$ *is assumed*

Strengths & Weaknesses of the Unified System

Strengths:

- Filters vertically propagating sound waves
- Does not need a basic or reference or mean state
- Is as accurate as the fully compressible system for non-acoustic modes
- Is easy to implement into an existing quasi-static model
- Can easily be “switched” to the quasi-static system
- Conserves energy

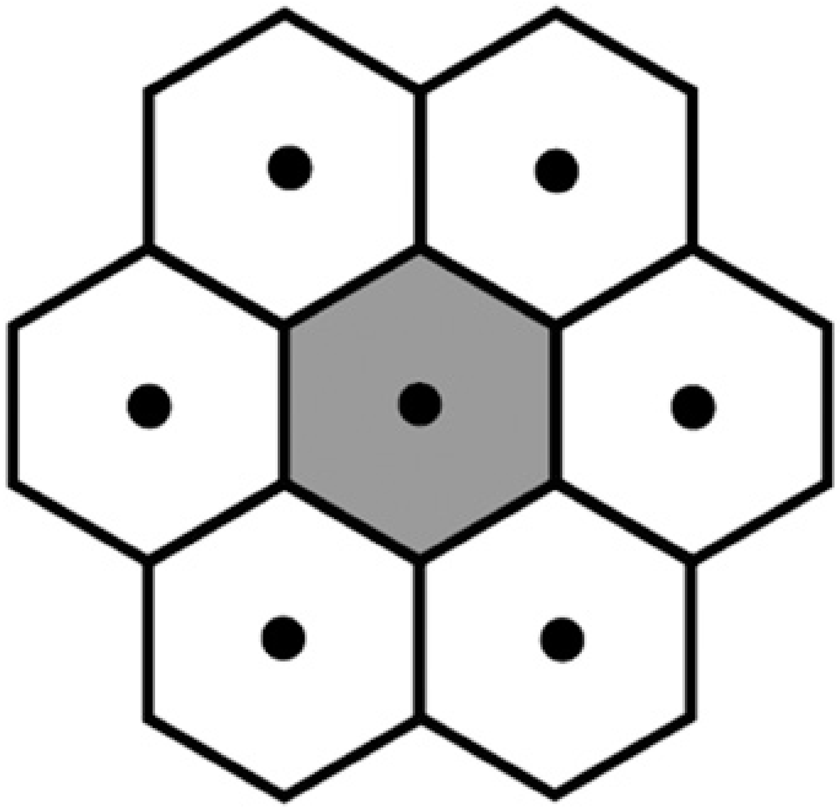
Weaknesses:

- Requires solution of a three-dimensional elliptic system

Choice of prognostic variables

With the continuous Unified System, there are two degrees of freedom in the wind for each degree of freedom in the mass field.

u, v, h



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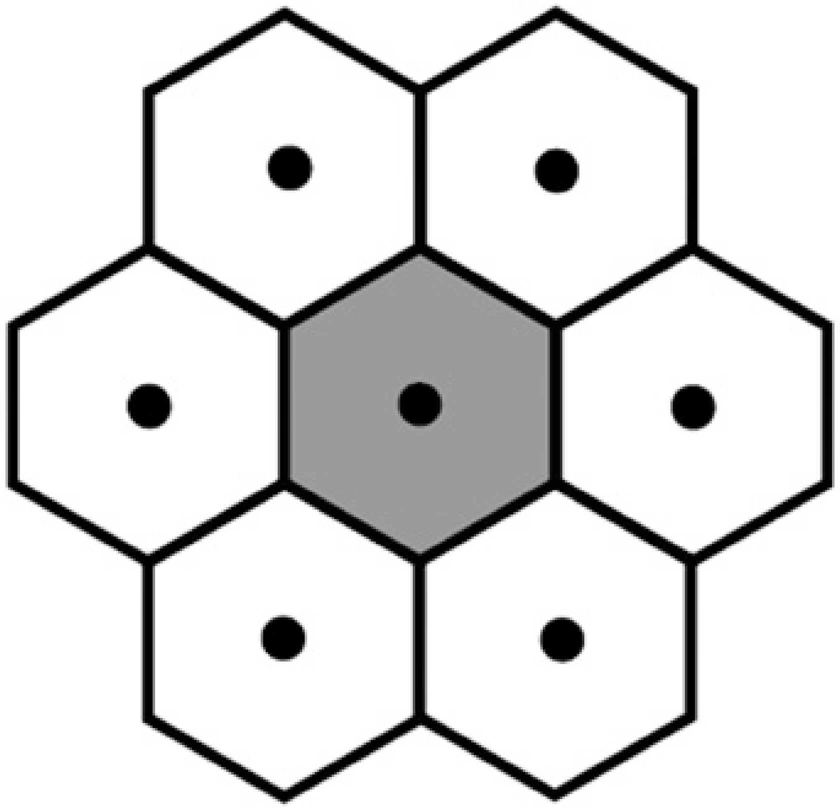
On geodesic C grids, the horizontal wind field has three degrees of freedom for each degree of freedom in the mass field.



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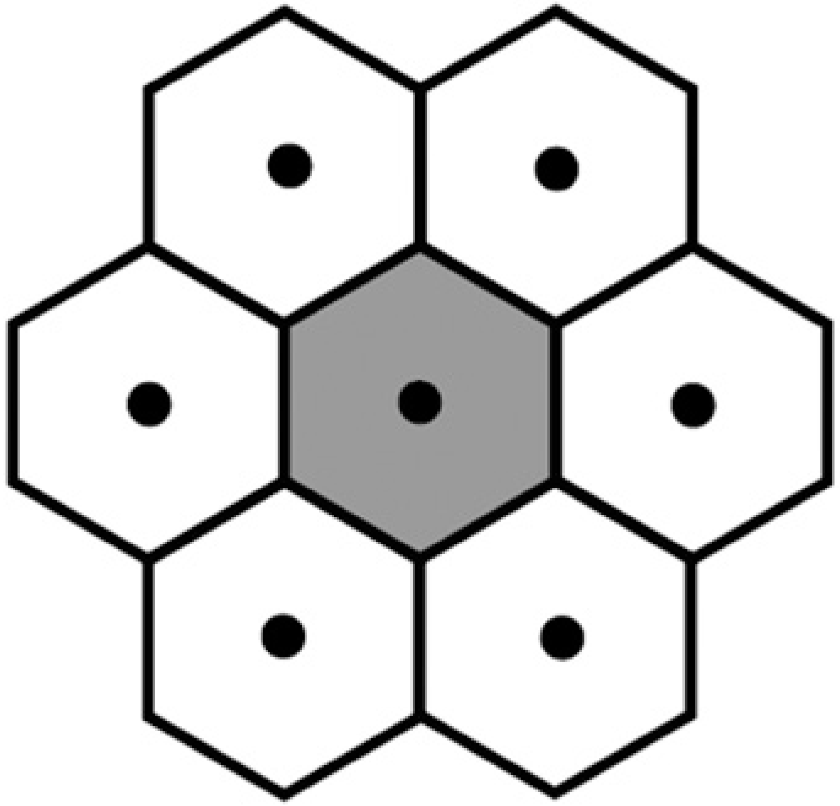

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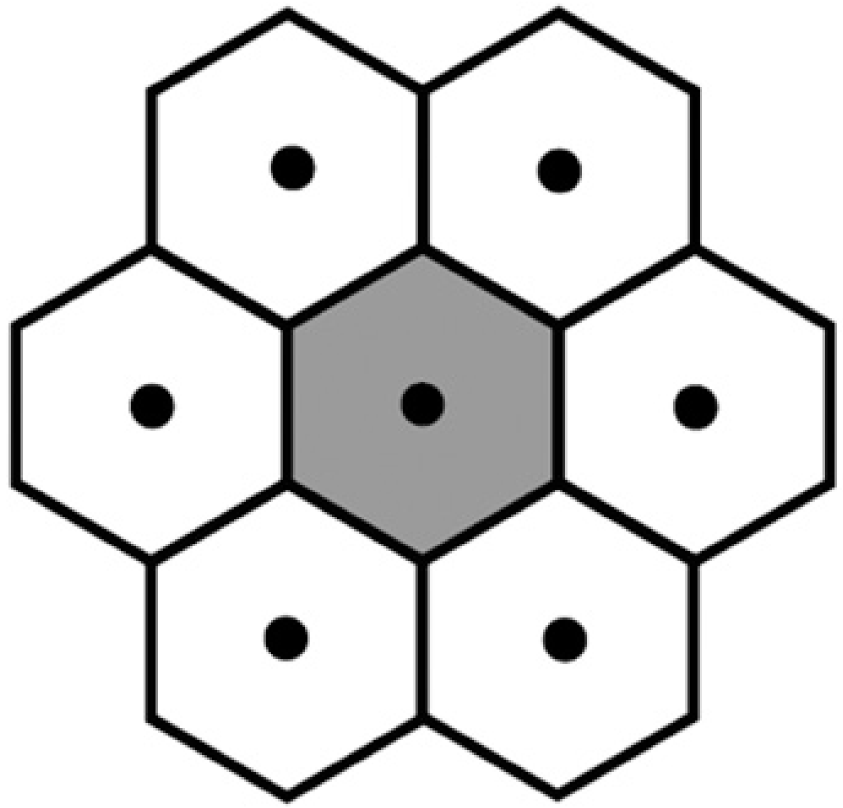

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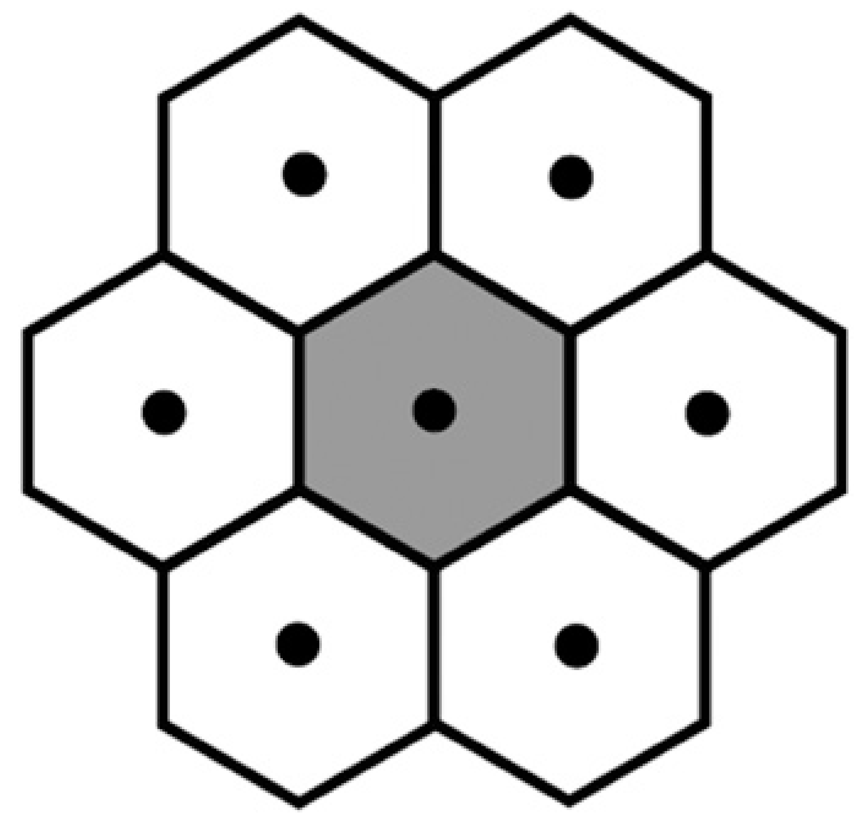

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The unstaggered A grid also suffers from computational modes, for a different reason.

The unstaggered Z grid has no computational modes.

ω_z, δ, h

Strengths & Weaknesses of the Z-grid

Strengths:

- No computational modes because matched degrees of freedom
- Prognostic pseudo-scalars, rather than vectors
- Excellent dispersion properties for inertia-gravity waves
- Direct prediction of the vertical component of the vorticity

Weaknesses:

- Requires solution of a pair of two-dimensional elliptic equations at each level on each time step

Elliptic equation for the pressure

When the Z-grid model is combined with the Unified System, it leads to a three-dimensional elliptic equation for the pressure:

$$\nabla_H \cdot \left(\rho_{qs} c_p \theta \nabla_H \delta\pi\right) + \frac{\partial}{\partial z}\left(\rho_{qs} c_p \theta \frac{\partial \delta\pi}{\partial z}\right) = forcing$$

$$\left(\frac{\partial \delta\pi}{\partial z}\right)_S = \left(\frac{\partial \delta\pi}{\partial z}\right)_T = 0$$

Neumann boundary conditions

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Neumann boundary conditions

Convergence is slow with the Neumann boundary conditions.

Prediction of vorticity allows us to escape this problem.

Vorticity across scales

Large-scale motions are controlled by the vertical component of the vorticity.

Small-scale motions are controlled by the horizontal vorticity vector.

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Small-scale motions are controlled by the horizontal vorticity vector.

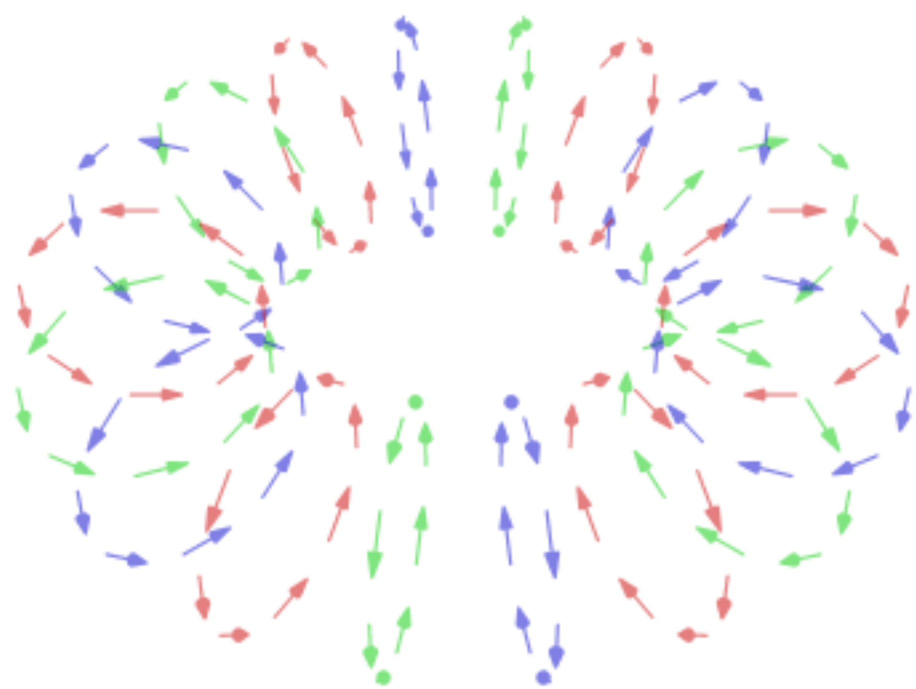
Realistic simulation of the vorticity is key on both large and small scales.

This motivates to predict the vorticity directly.

The wind field can be "reconstructed" by integration.

Vector Vorticity Model (VVM)

Jung & Arakawa, 2008



The VVM predicts the
horizontal vorticity vector
on a C-grid

$$\boldsymbol{\omega}_{\mathrm{H}} = \frac{\partial \mathbf{V}_{\mathrm{H}}}{\partial z} - \nabla_{\mathrm{H}} w$$

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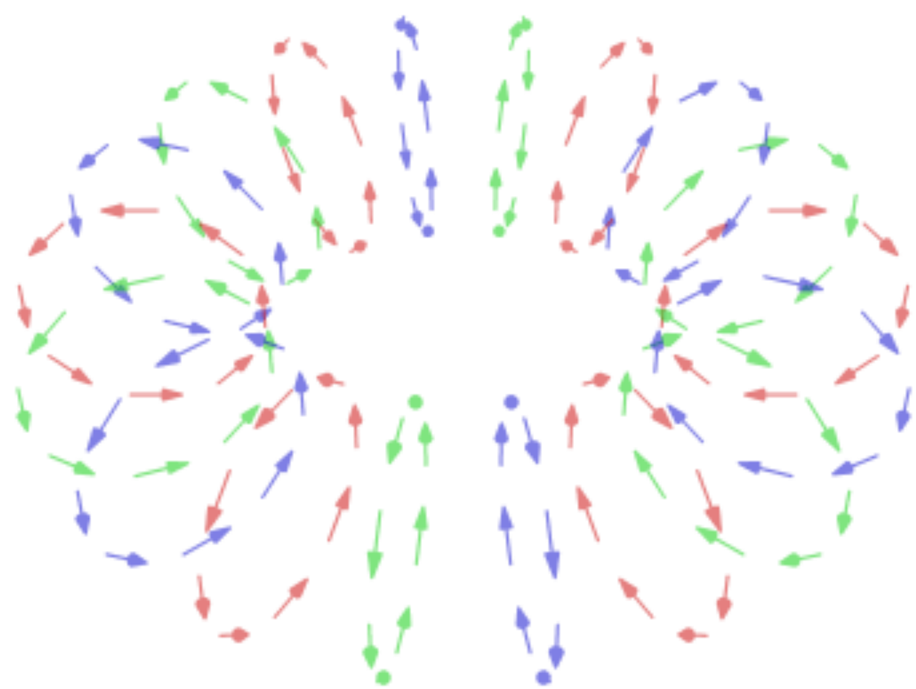

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Elliptic equation for the vertical velocity

With the VVM, a three-dimensional elliptic equation is solved for the vertical velocity, rather than the pressure:

$$\nabla_{\mathrm{H}}^2 w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_{qs}}\frac{\partial}{\partial z}\left(\rho_{qs} w\right)\right] = -\Gamma \equiv \mathbf{k} \cdot \nabla_{\mathrm{H}} \times \boldsymbol{\omega}_{\mathrm{H}}$$

$$w_S = w_T = 0$$

Dirichlet boundary conditions

Convergence is fast.

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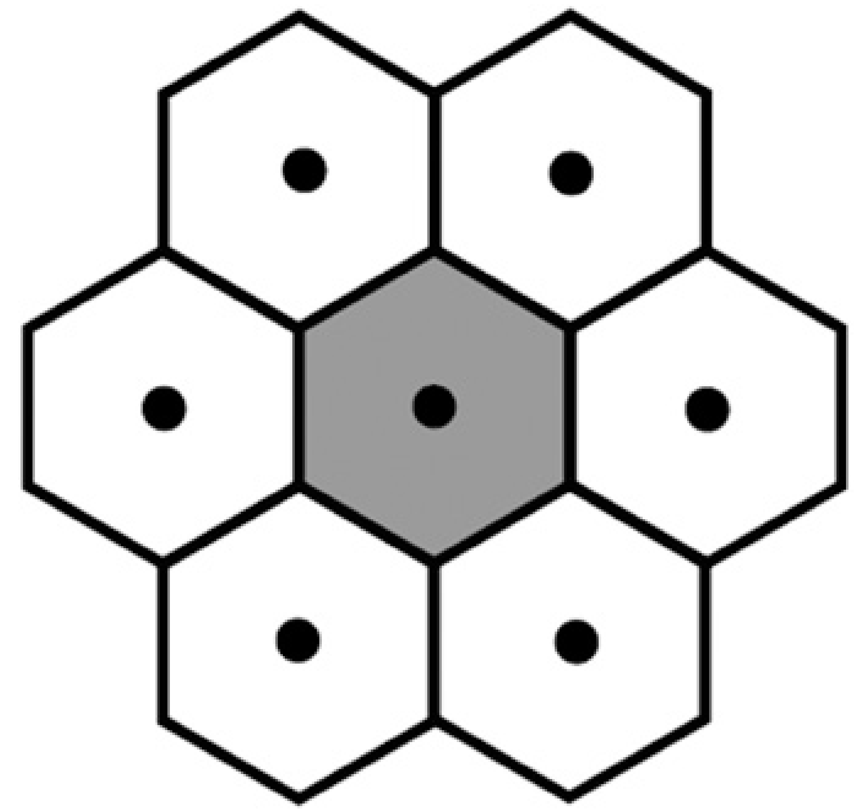
Dirichlet boundary conditions

Convergence is fast.

The "forcing" on the right-hand side is the curl of the horizontal vorticity.

A geodesic VVM?

The VVM predicts the tangential component of the horizontal vorticity on each cell wall. It uses the C grid.



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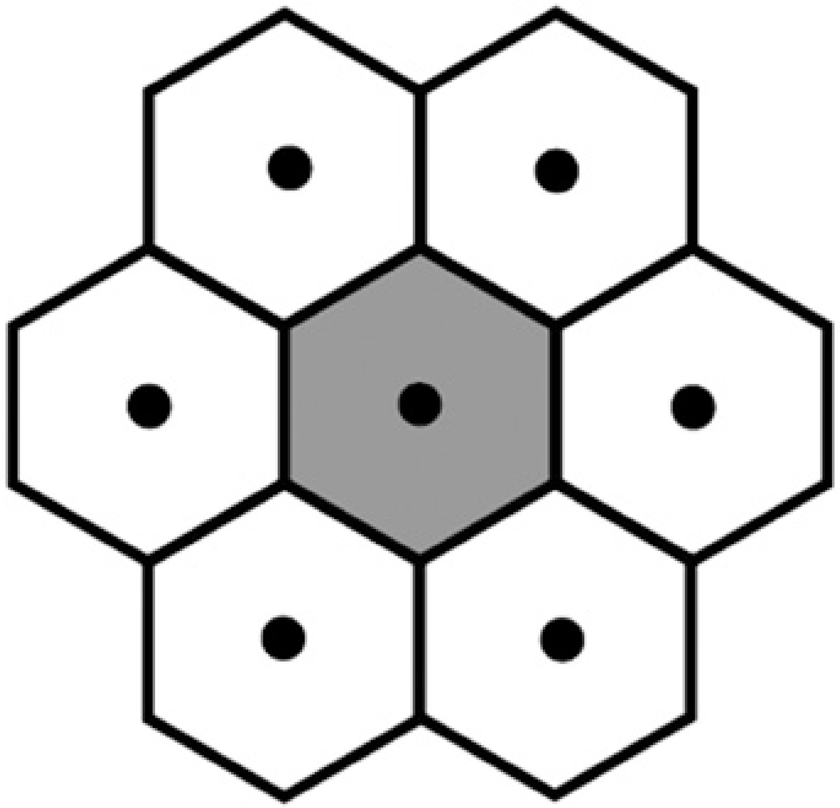

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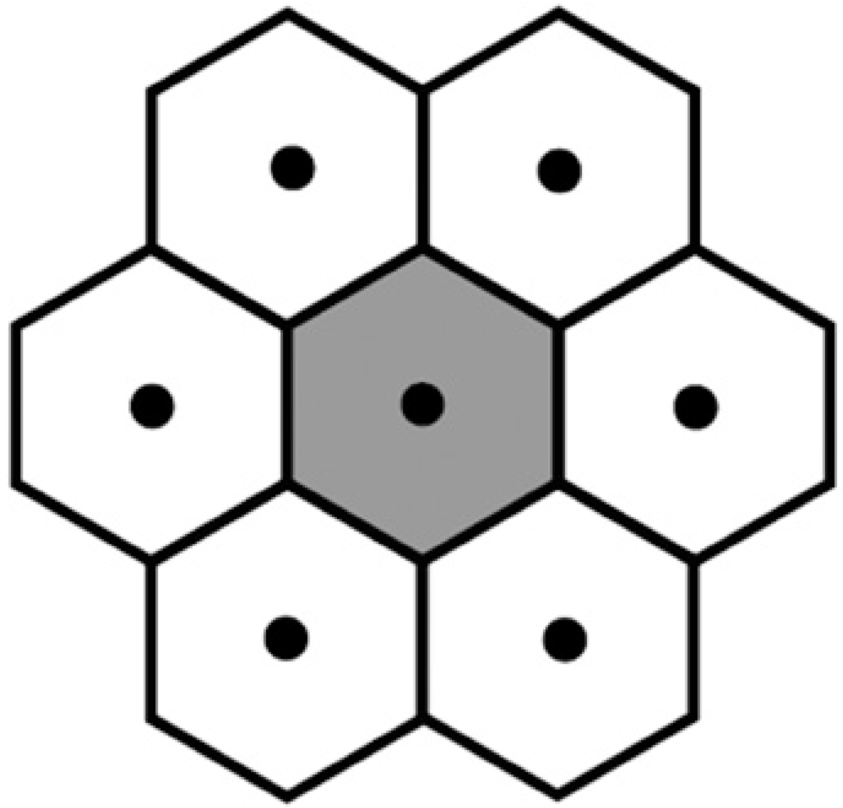

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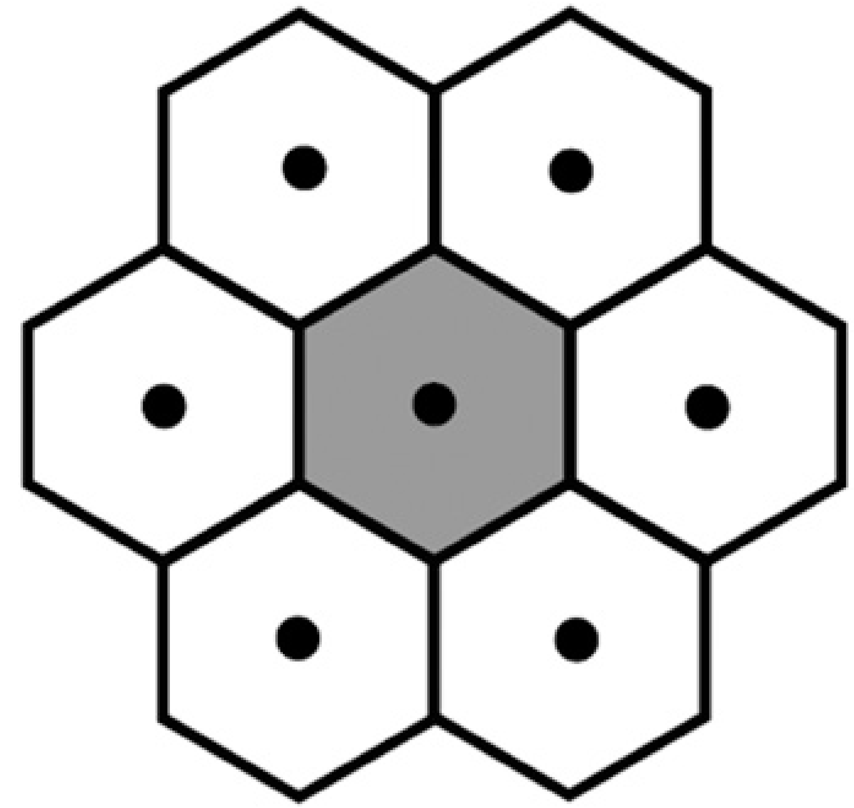

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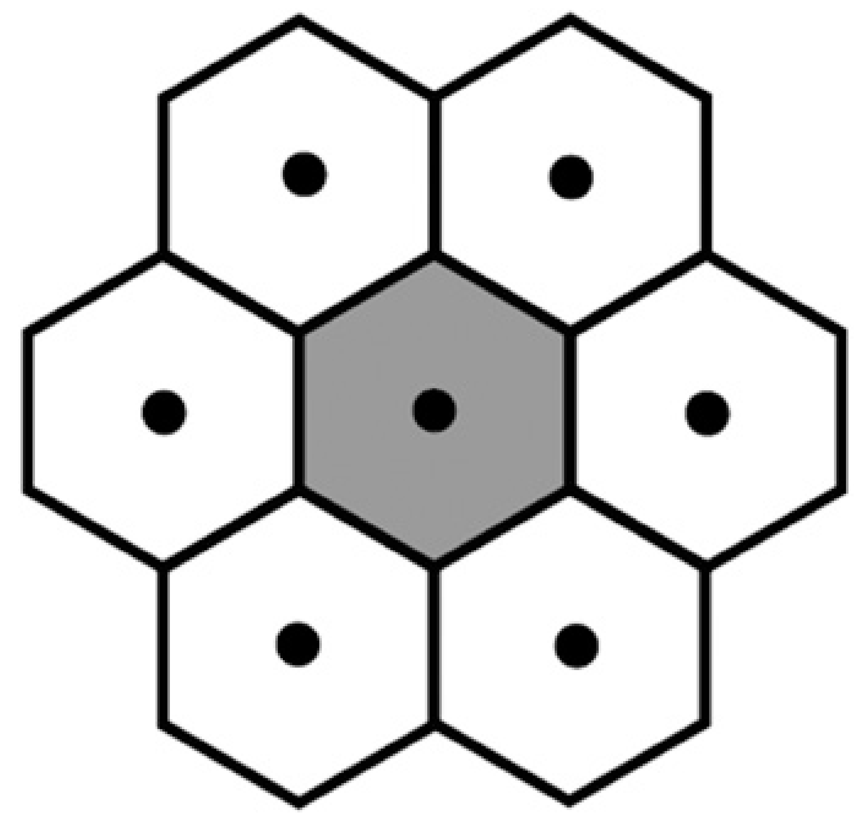
We will use the same solution.



Curl Curl

Z-grid model	Curl Curl
$\omega_z \equiv \mathbf{k} \cdot \left(\nabla_{\mathrm{H}} \times \mathbf{V}_{\mathrm{H}} \right)$	$\Gamma \equiv \mathbf{k} \cdot \left(\nabla_{\mathrm{H}} \times \boldsymbol{\omega}_{\mathrm{H}} \right)$
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Since $\nabla_3 \cdot \boldsymbol{\omega}_3 = 0$ we can write $\nabla_H \cdot \boldsymbol{\omega}_H = -\frac{\partial \omega_z}{\partial z}$.

So, what we actually predict are ω_z and $\Gamma \equiv \mathbf{k} \cdot (\nabla_H \times \boldsymbol{\omega}_H)$.

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What is the curl of the vorticity?

The vorticity has a curl when the vortex lines make loops or rings, analogous to the circular structures sometimes formed by the wind field when the velocity has a curl.

Vortex loops or rings surround jets, plumes and thermals.

The horizontal vorticity also has a curl in a field of cloud streets or "rolls," because in that situation the horizontal vorticity changes sign in the direction perpendicular to the vorticity vector.

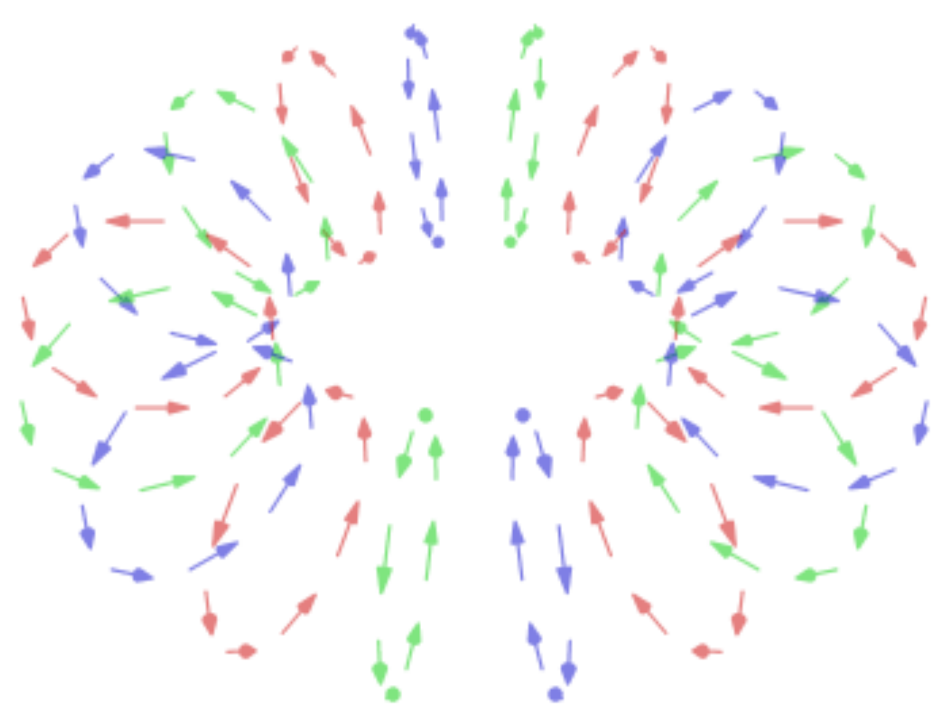
The Gamma Equation

$$\frac{\partial \Gamma}{\partial t} - \nabla_r^2 d - \frac{1}{r}\frac{\partial}{\partial r}\left\{ r\mathbf{e}_r \cdot \nabla_r \times \left[\mathbf{V}_h \left(\zeta + 2\Omega_r \right) - w\left(\boldsymbol{\eta} + 2\boldsymbol{\Omega}_h \right) \right] \right\}$$

$$= \nabla_r \cdot \left(-\frac{\partial \alpha}{\partial r} \nabla_r p + \frac{\partial p}{\partial r} \nabla_r \alpha \right) - \mathbf{e}_r \cdot \nabla_r \times \left(\nabla \times \mathbf{F} \right)_h$$

where

$$d\mathbf{e}_r \equiv \left(\boldsymbol{\eta} + 2\boldsymbol{\Omega}_h \right) \times \mathbf{V}_h$$



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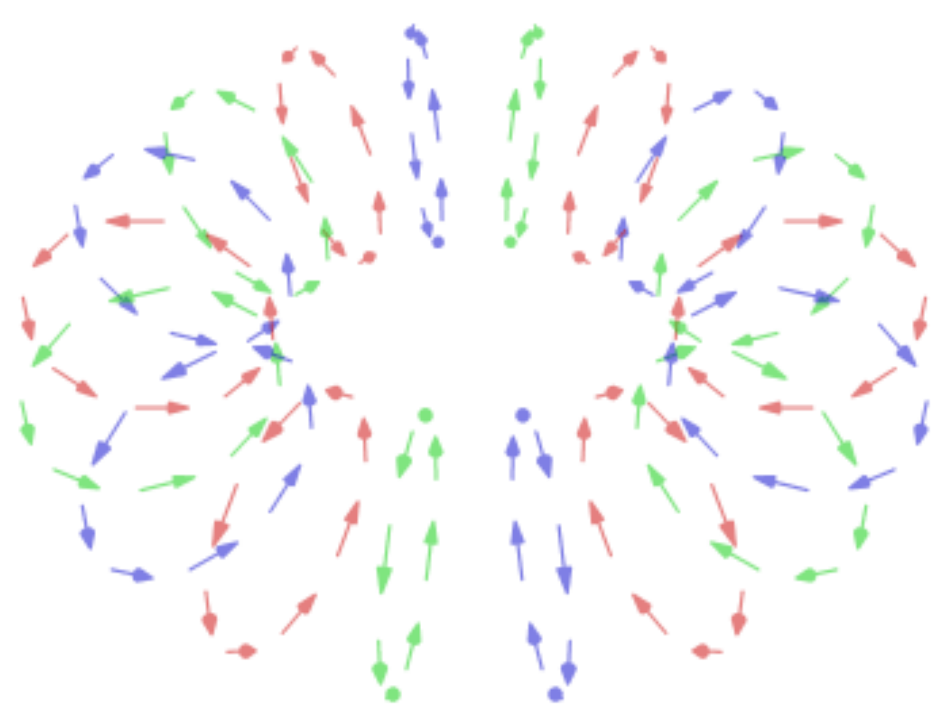
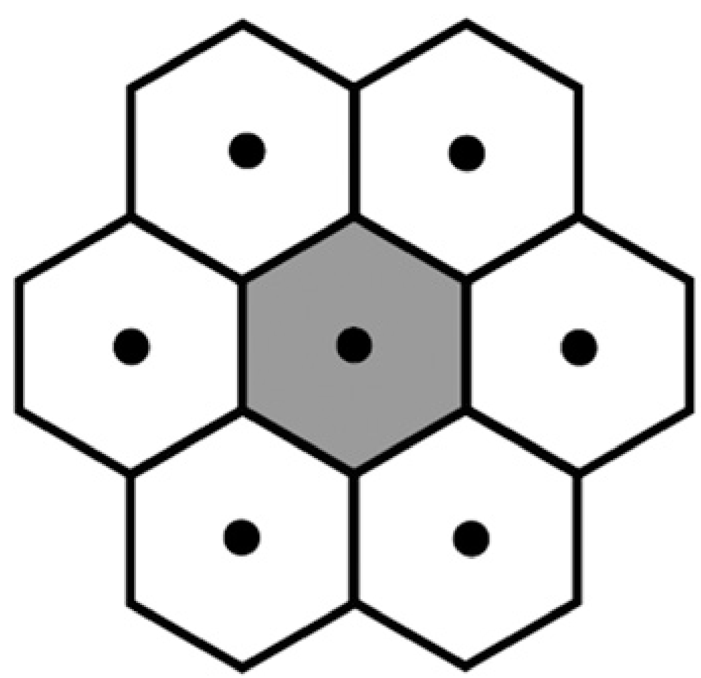


How it works

Starting from $\nabla_{\mathrm{H}} \cdot \boldsymbol{\omega}_{\mathrm{H}} = -\dfrac{\partial \omega_z}{\partial z}$ and $\Gamma \equiv \mathbf{k} \cdot \left(\nabla_{\mathrm{H}} \times \boldsymbol{\omega}_{\mathrm{H}} \right)$,

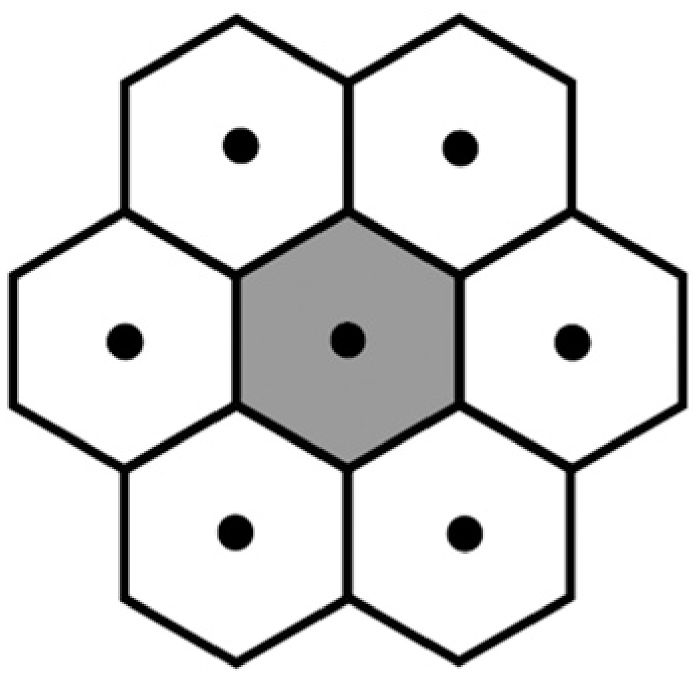
we can diagnose $\boldsymbol{\omega}_{\mathrm{H}}$ by solving a pair of elliptic equations, just like we do with the Z-grid model.

Once $\boldsymbol{\omega}_{\mathrm{H}}$ has been determined, the logic follows the VVM exactly.



Unstaggered grids

u, v, h



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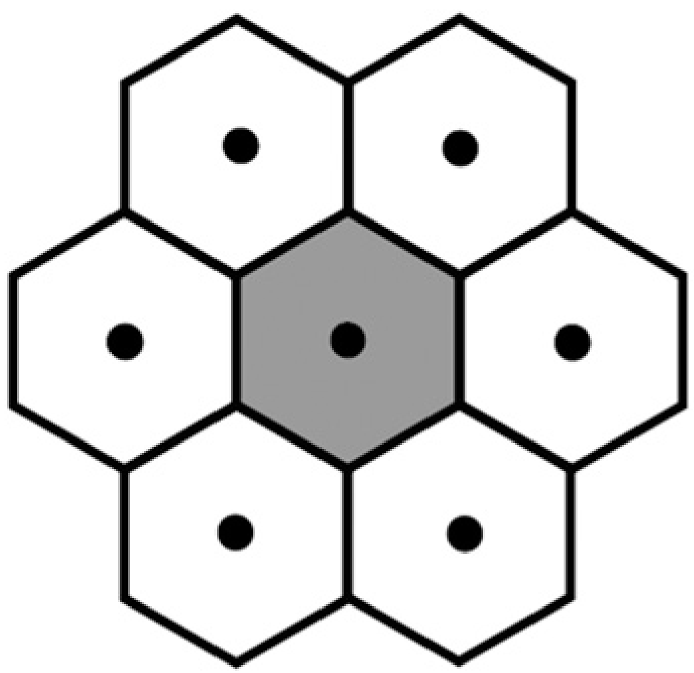

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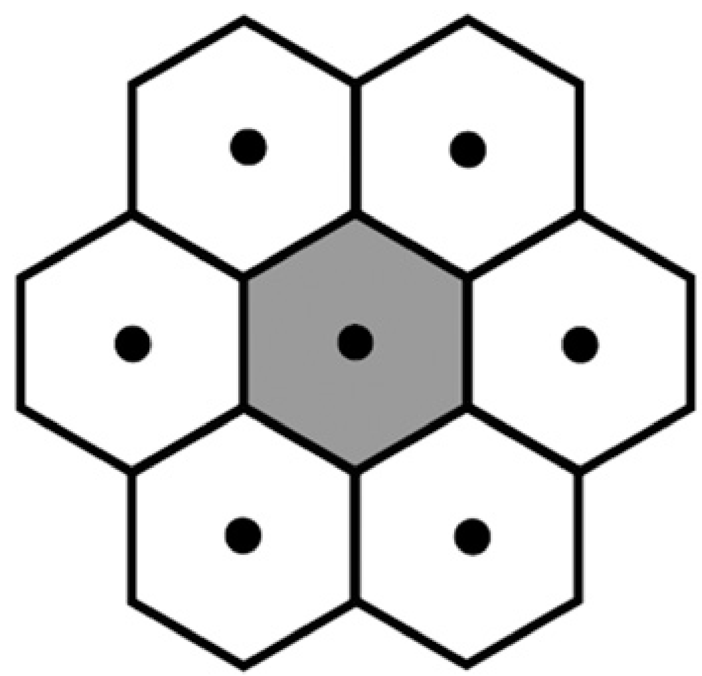
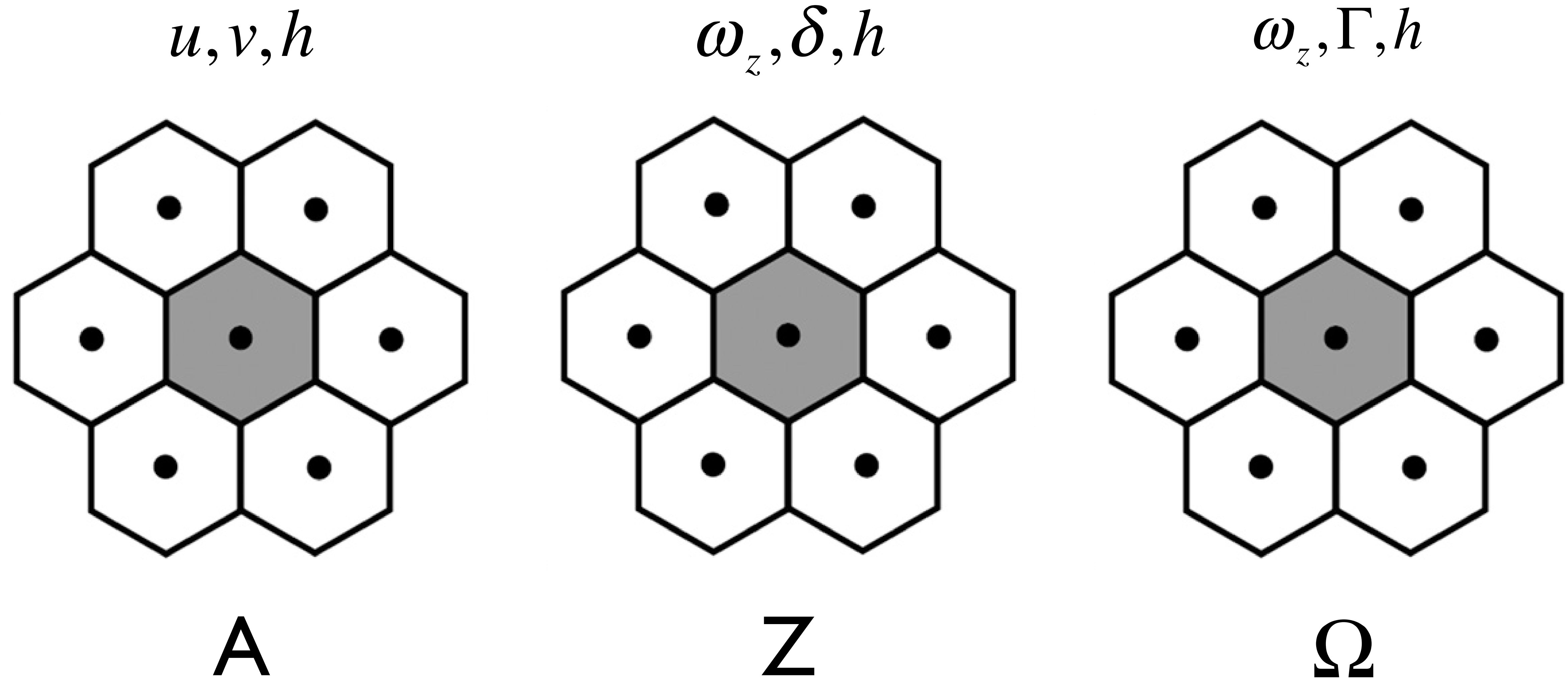
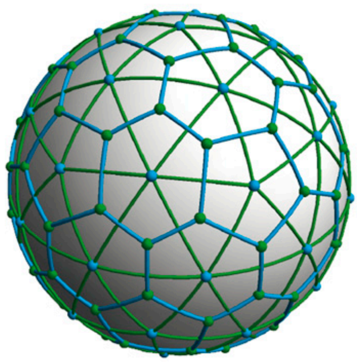


Z

Unstaggered grids



Curl Curl =



Geodesic grid

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Unified system

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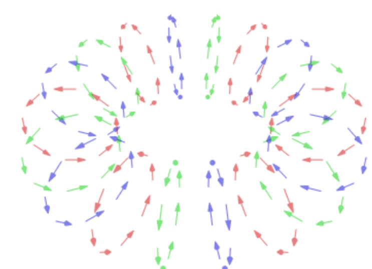

Vector vorticity

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Ω

Ω grid

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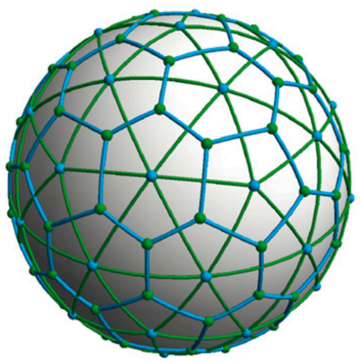

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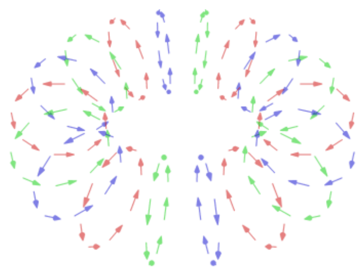

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Ω

Geodesic grid

Unified system

Vector vorticity

Ω grid

Strengths & Weaknesses of Curl Curl

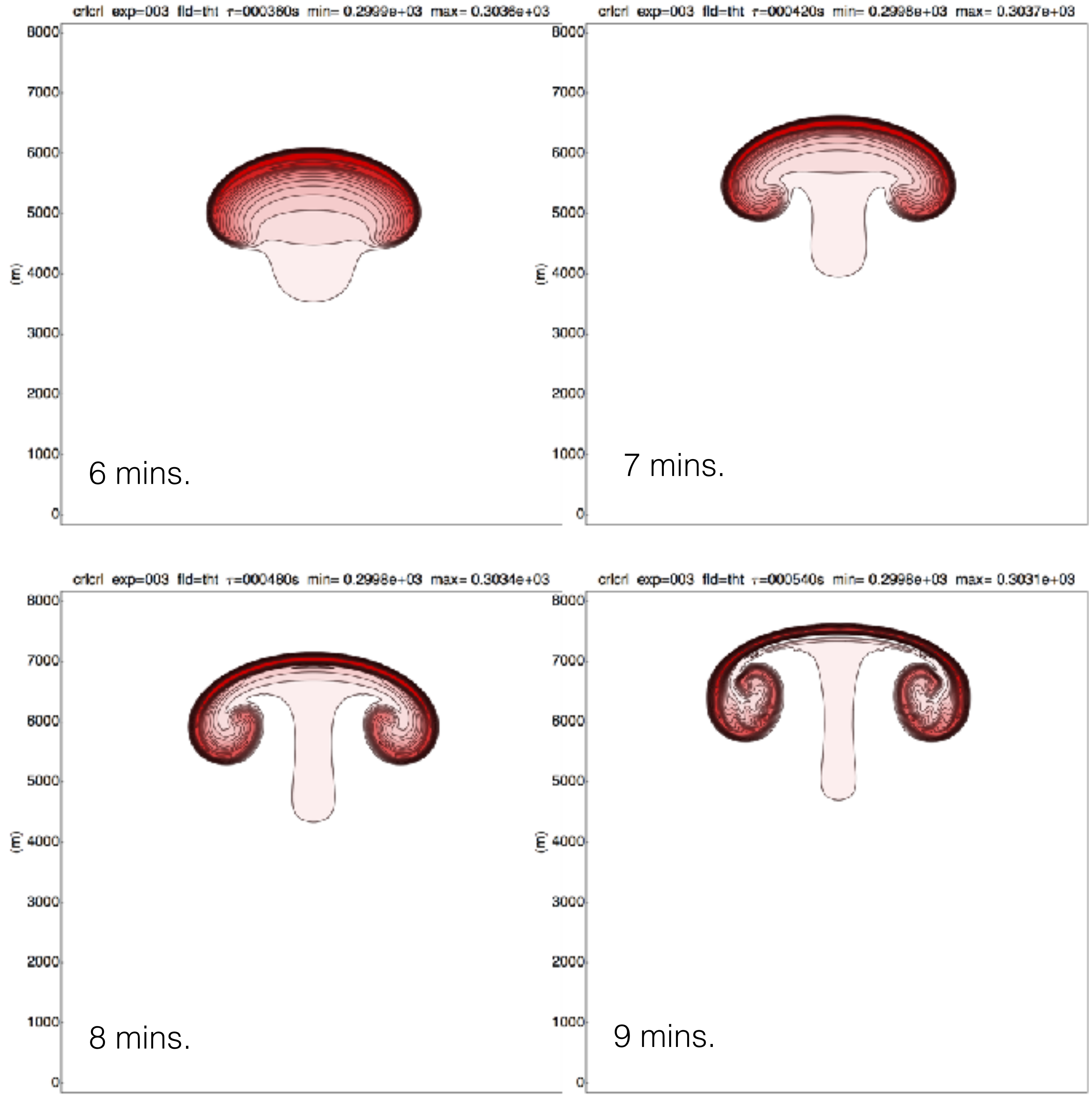
Strengths:

- No computational modes (because Ω grid)
- Excellent dispersion properties for inertia-gravity waves (because Ω grid)
- Direct prediction of the vertical component of the vorticity, which controls large-scale dynamics
- Direct prediction of the curl of the horizontal vorticity, which controls small-scale dynamics
- Predicts pseudo-scalars, rather than vectors (because Ω grid)
- Guarantees the non-divergence of the three-dimensional vorticity vector (because VVM)

Weaknesses:

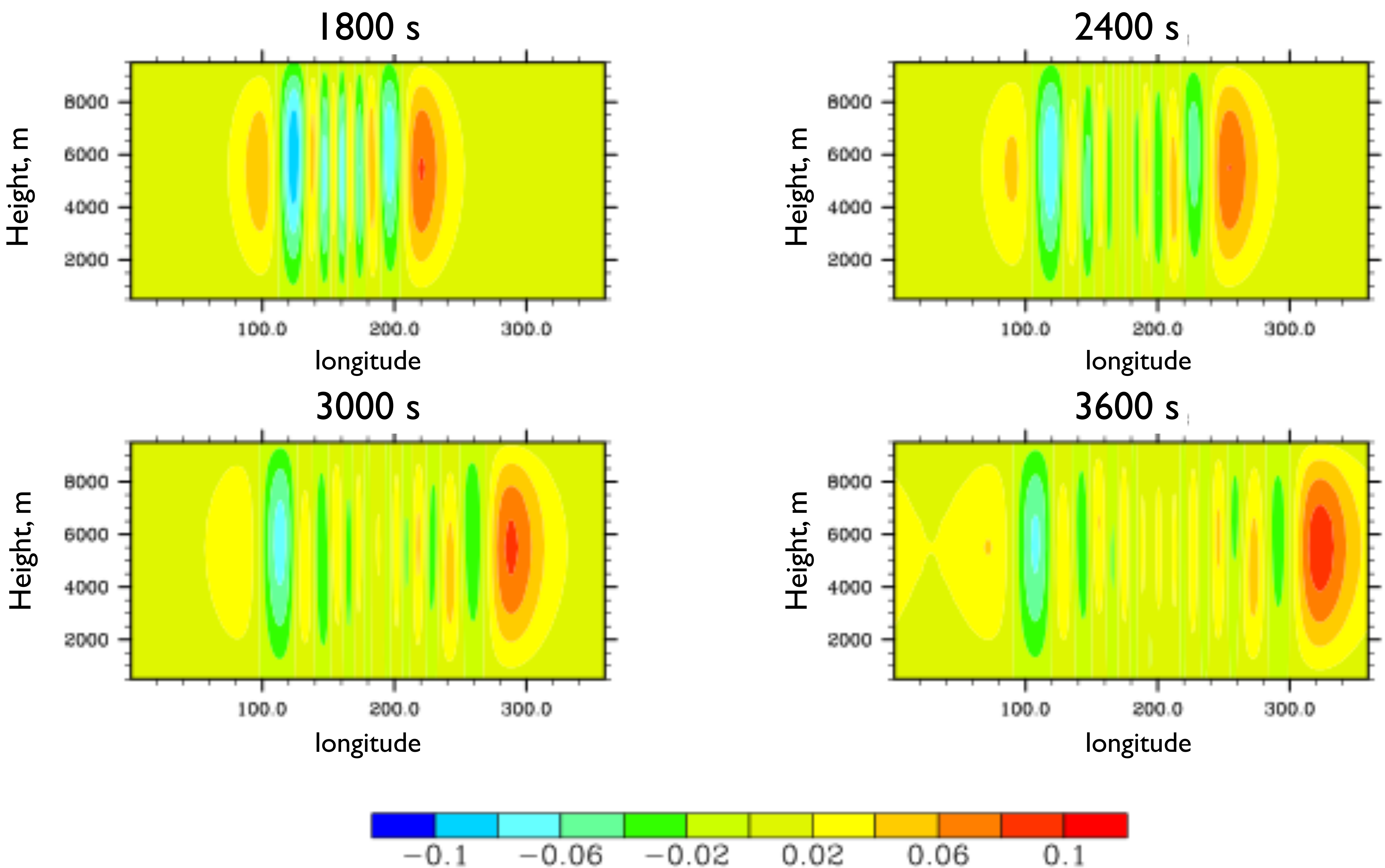
- Requires solution of a pair of 2D elliptic equations at each level on each time step
- Requires solution of a 3d elliptic equation, but with "friendly" Dirichlet boundary conditions

Anelastic test case.
Rising warm bubble.



ENDGame results

DCMIP test 3.1, theta prime along the Equator, K



Curl Curl results

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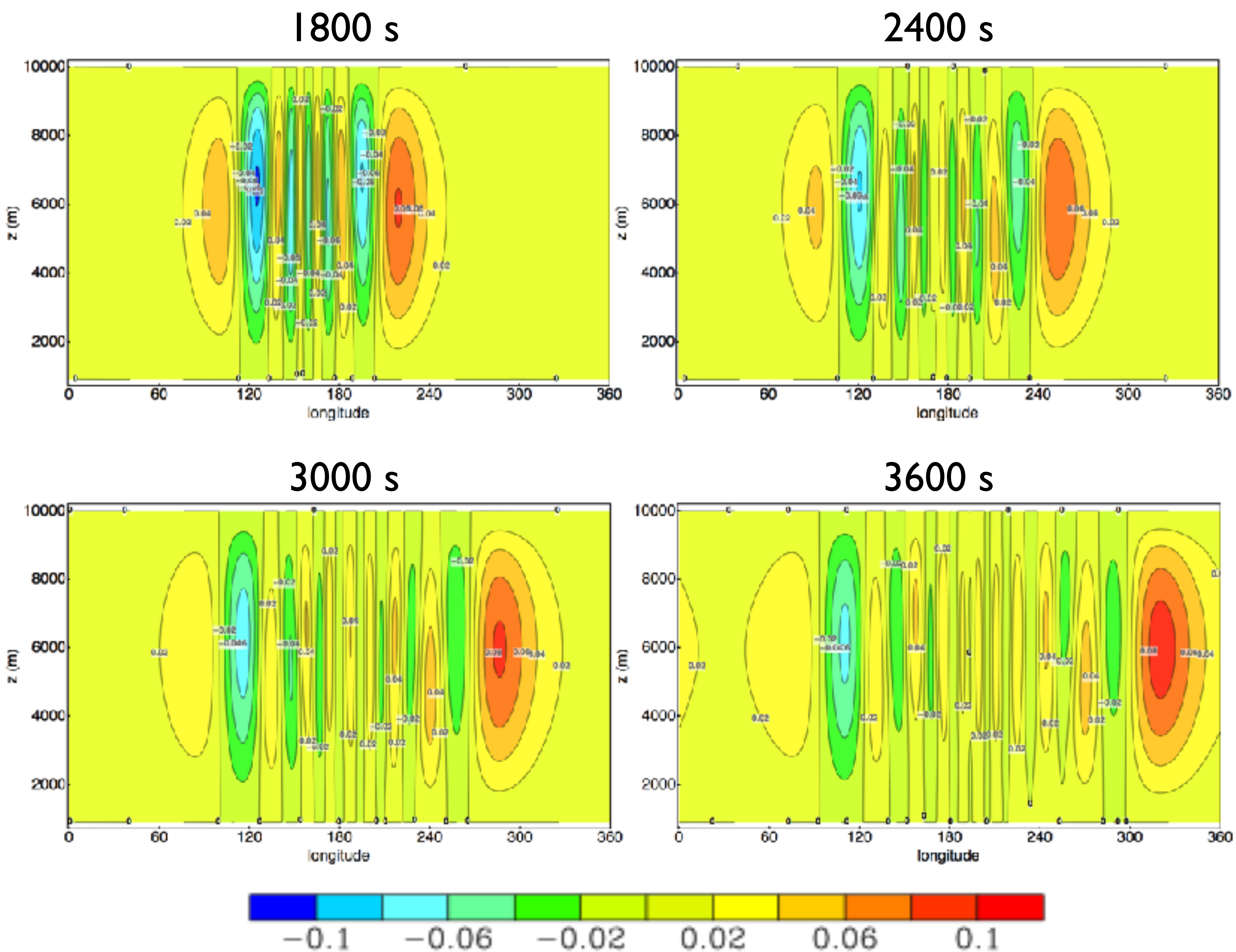
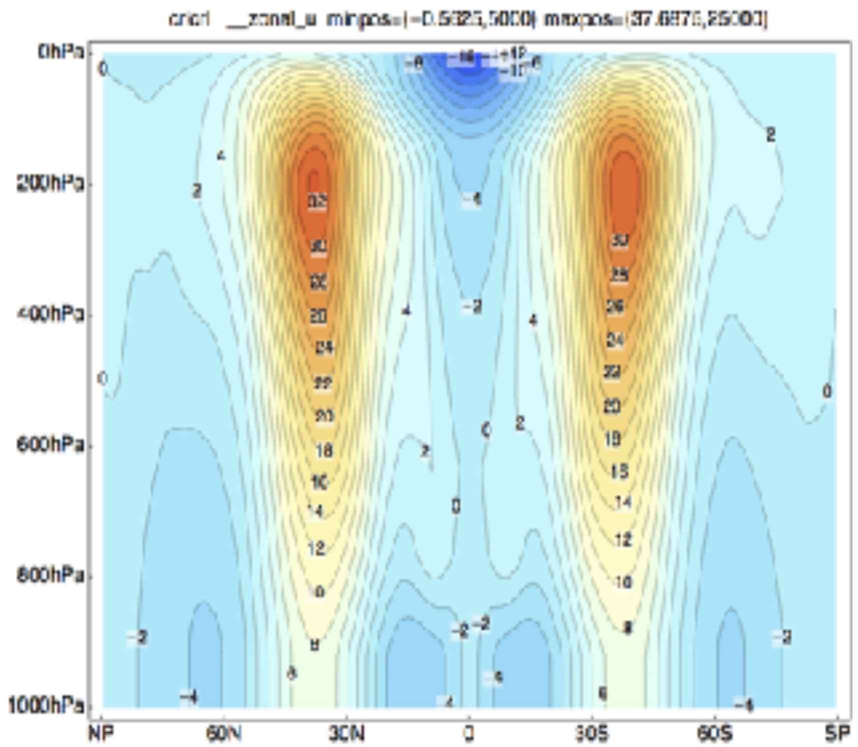


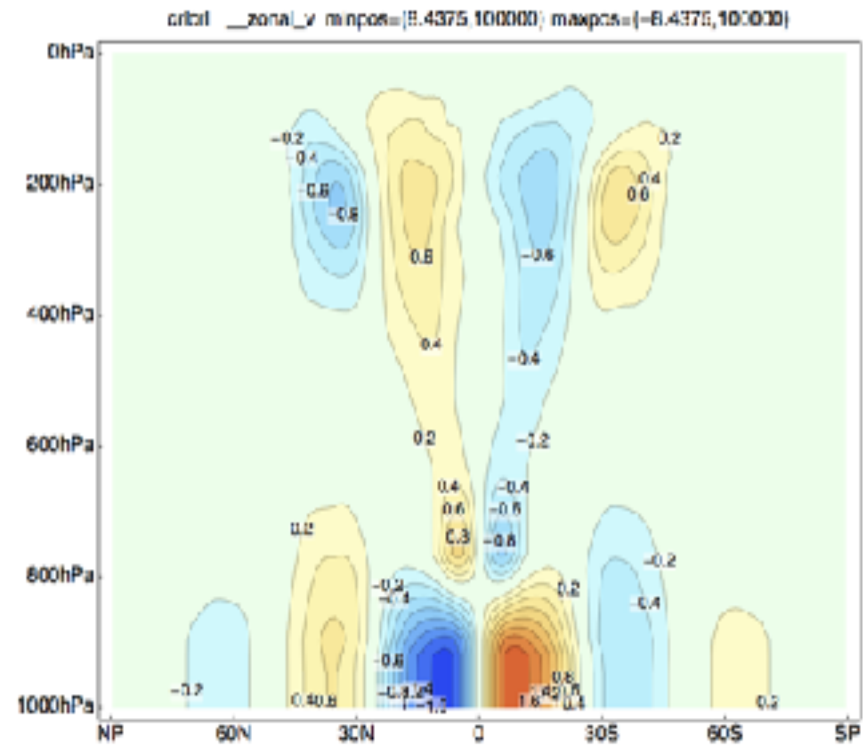
Held-Suarez test

Curl-Curl, 40 K grid, days 1000 to 2000

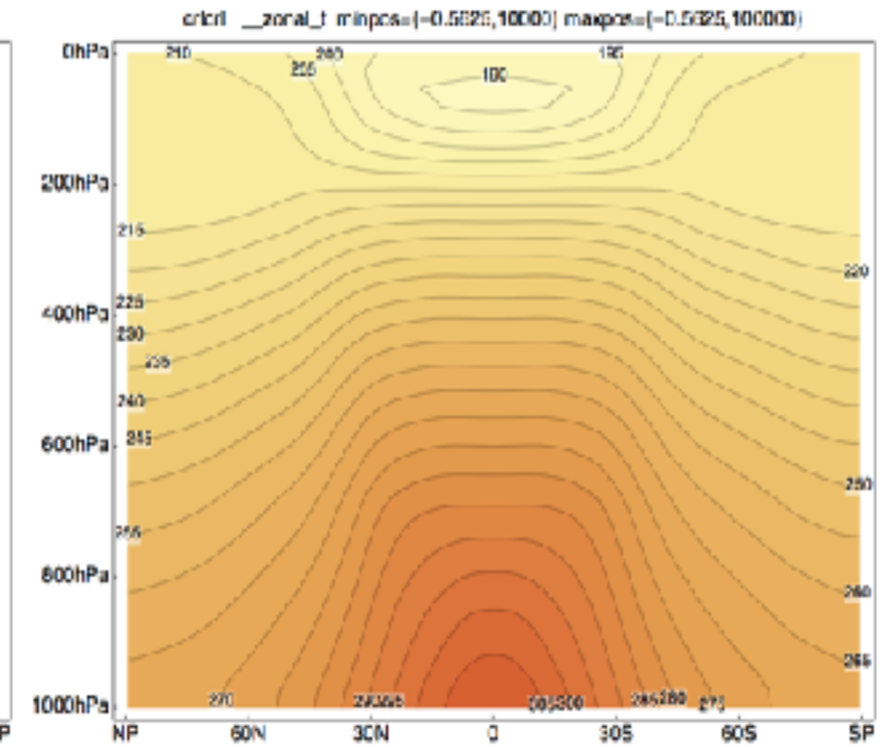
ubar



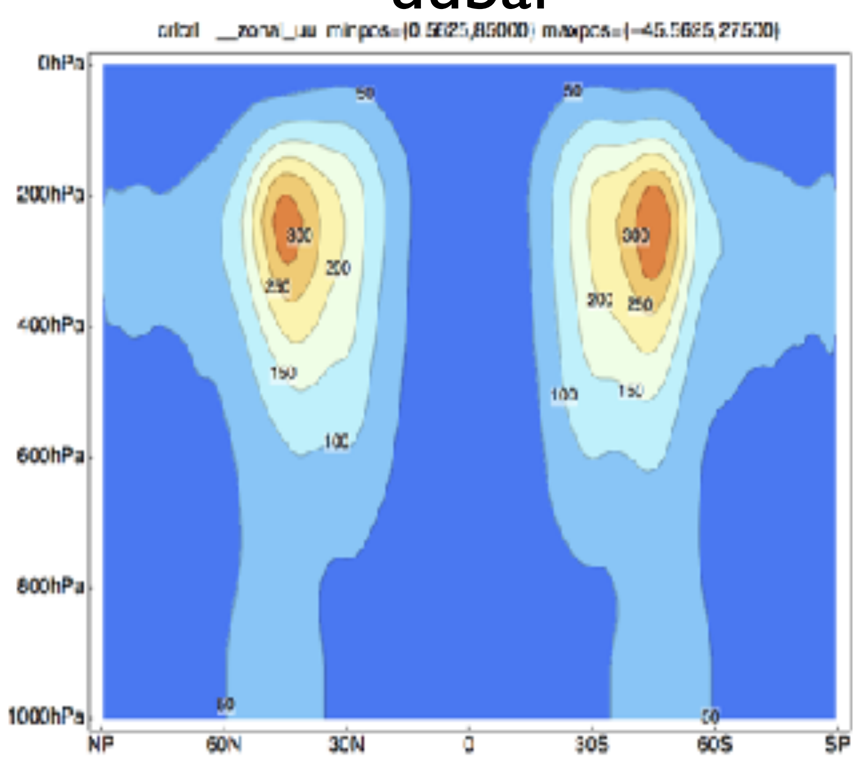
vbar



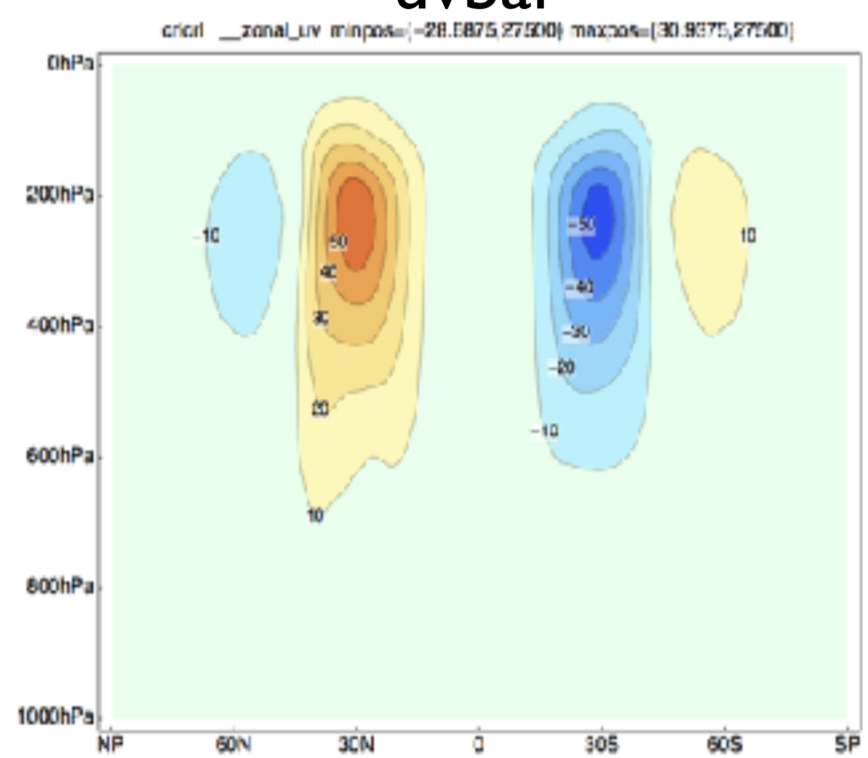
Tbar



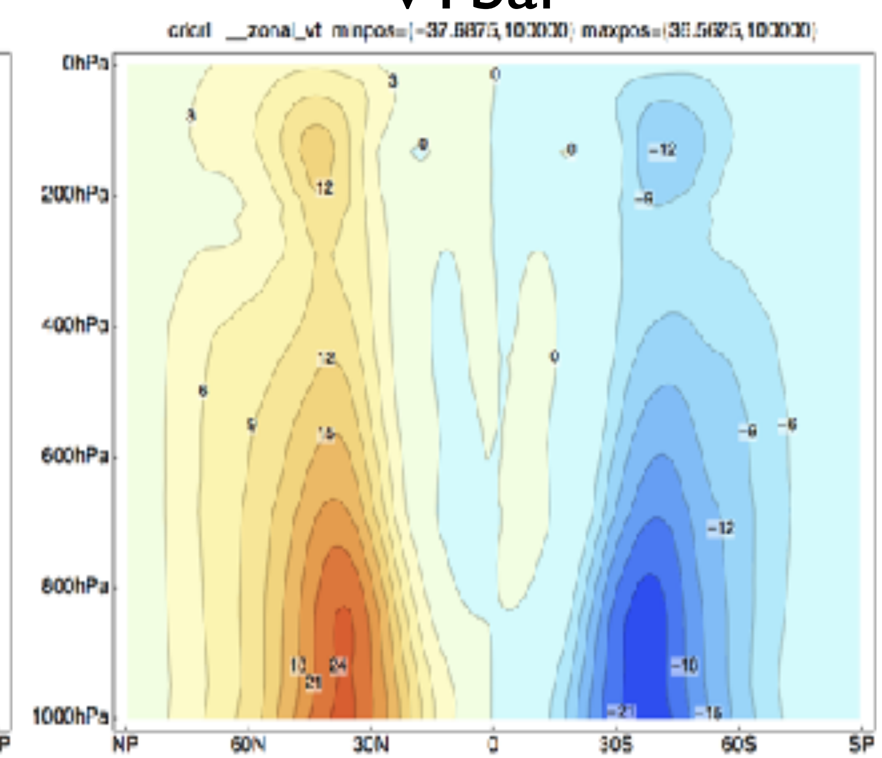
uubar



uvbar



vTbar



Closing Remarks

- This talk has been about structural issues.
- Structural design comes first. The ideas discussed in this talk form the concrete and steel of the model, on which everything else depends.
- The forms of the various operators also have to be specified, of course, and it's important to do a good job with that. Ross Heikes has developed some very accurate and flexible operators for use with Curl Curl.
- Computational performance and scaling are also important. Curl Curl scales well.

Extra slides

Ω grid

Relevant publications

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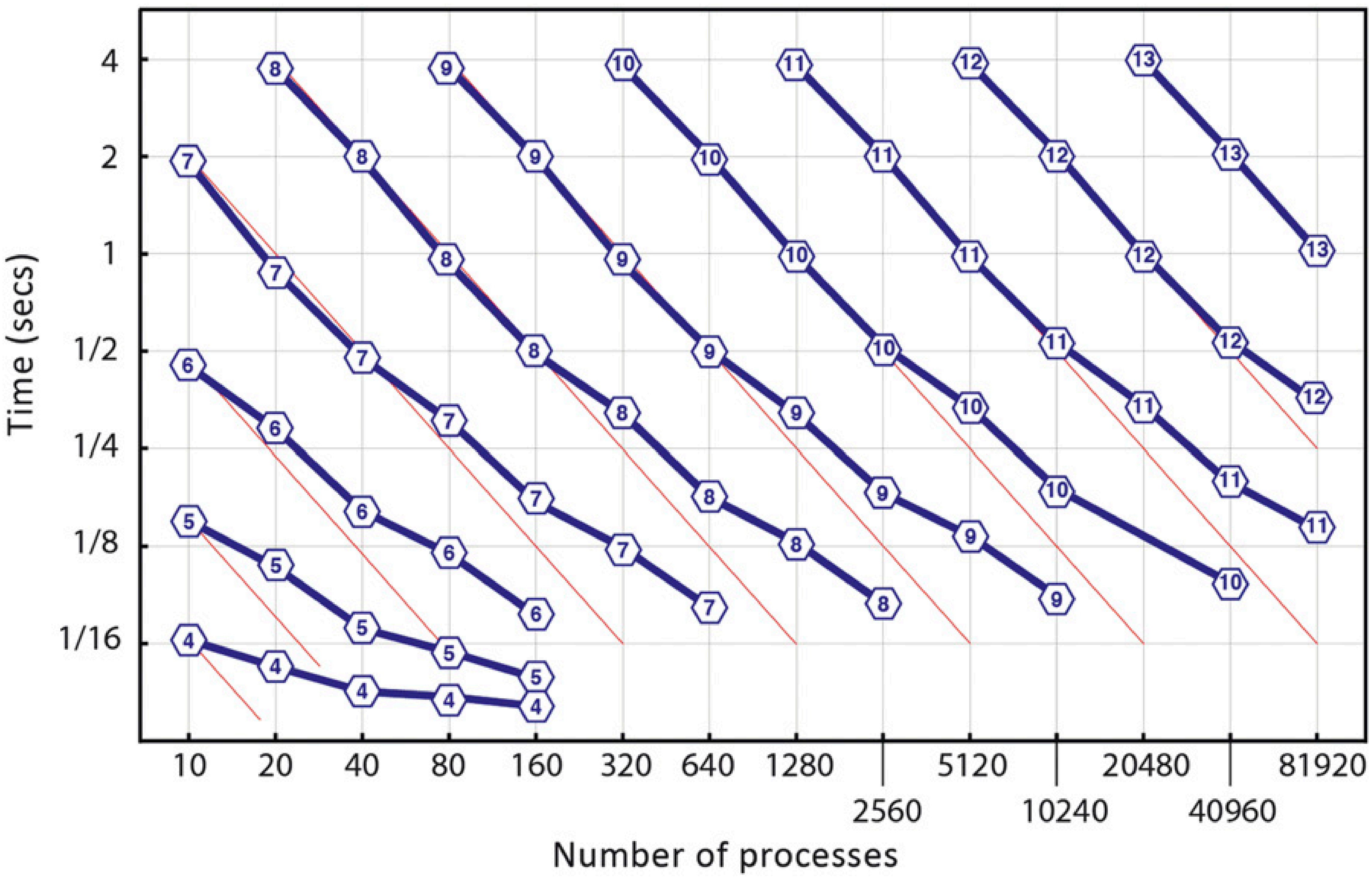
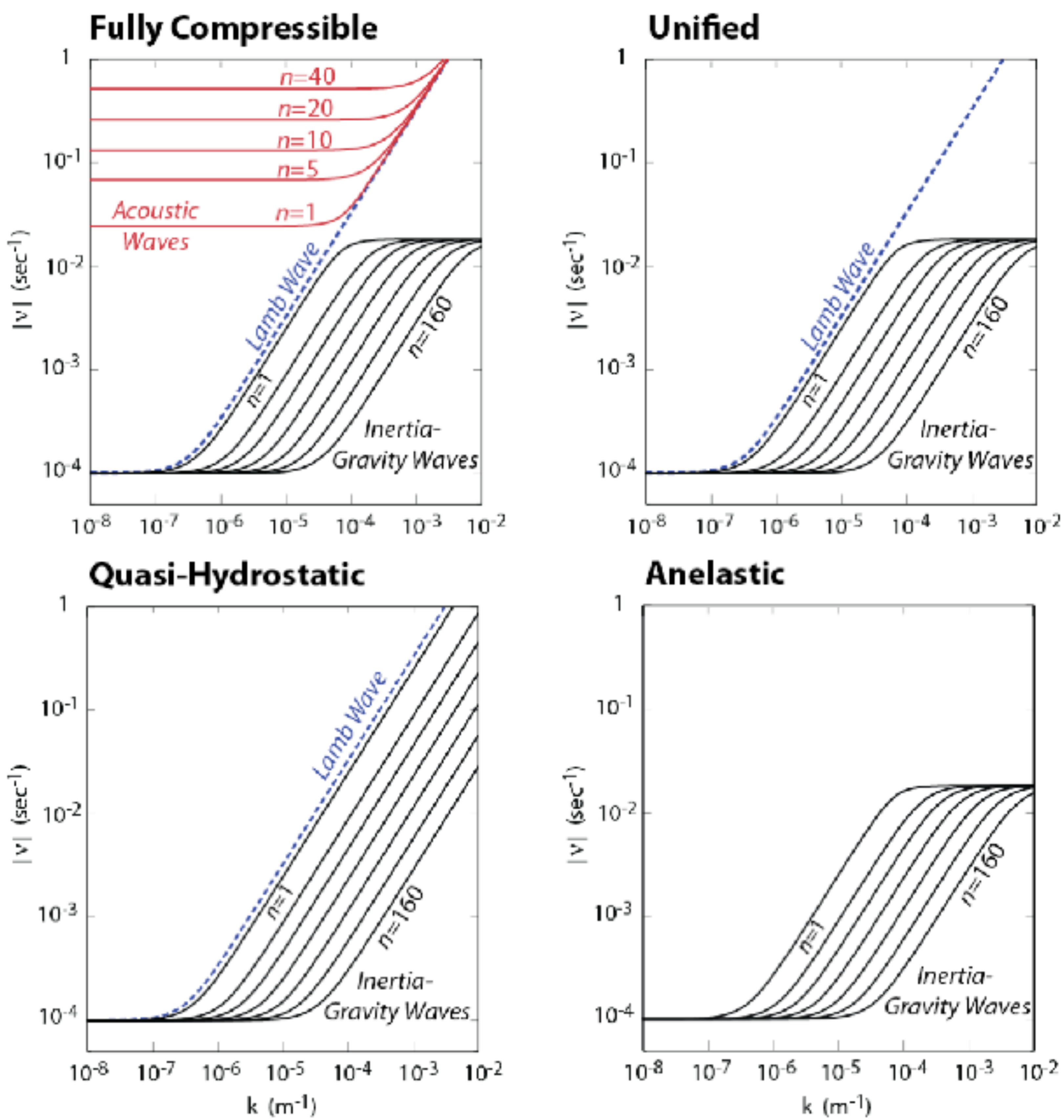
Multigrid scaling



Dispersion of Inertia-Gravity Waves



Dispersion of Rossby Waves