Topographic representation scheme using a thin-wall approximation on terrain-following coordinates

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1. Topographic schemes for atmospheric models

- Terrain-following coordinates is most popular in atmospheric models.
- Over steep topography, it induces the numerical errors.

e.g. Janjic (1989), Steppler et al. (2006, 2011), God et al.(2013)

The other approaches in z-coordinates:

- Step / block method

  e.g. Mesinger et al. (1988), Wu and Arakawa (2011),
  Tripoli and Smith (2014)

- Cut cell method

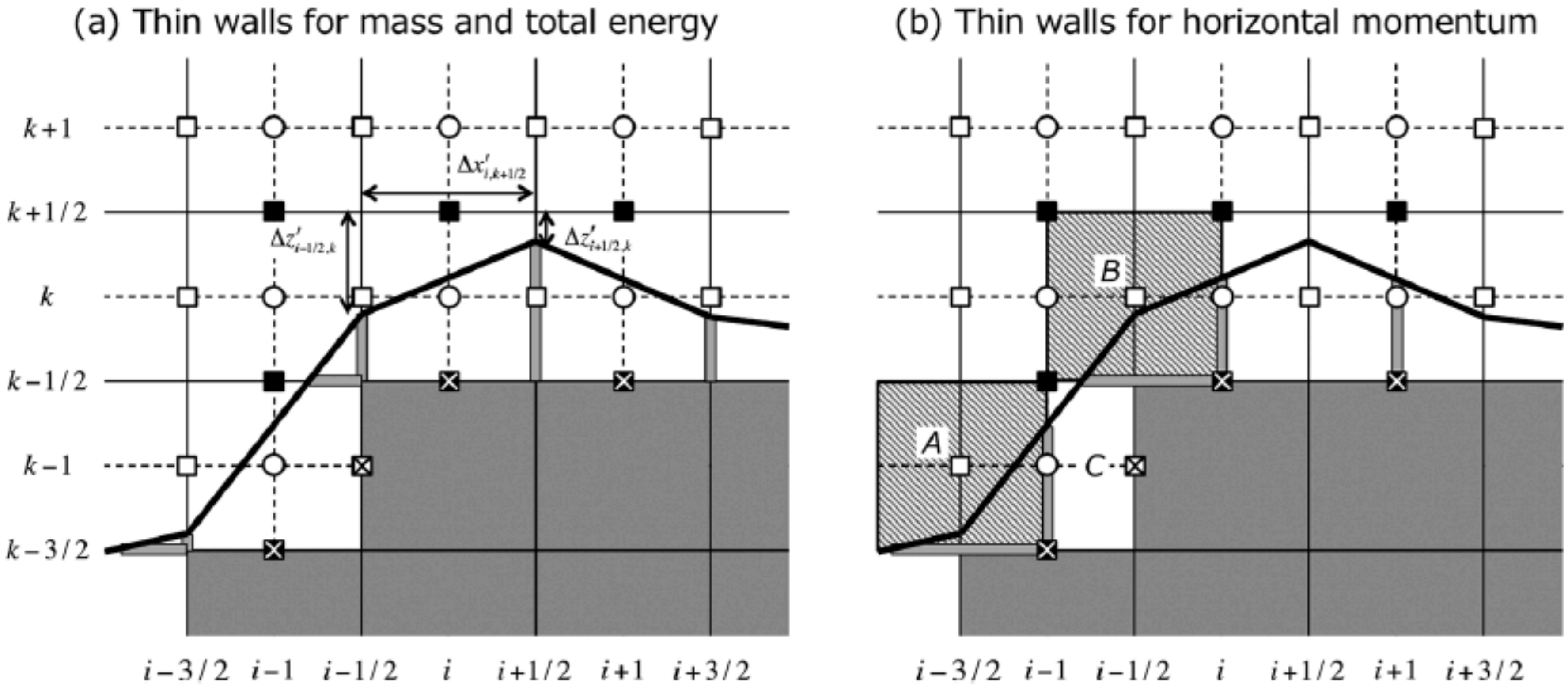
  e.g. Steppeler et al. (2002, 2006. 2011), Yamazaki and Satomura (2010), Lock et al. (2012), Nishikawa and Satoh (2016)

- Immersed boundary method

  e.g. Tseng and Ferziger (2003), Chien and Wu（2016）

2. Thin-wall method (Nishikawa and Satoh, 2016)

C-grid model on z-coordinates


(a) Thin walls for mass and total energy

(b) Thin walls for horizontal momentum

$$F_{zi-1/2,k} = \frac{\Delta z'_{i-1/2,k}}{\Delta z_{i-1/2,k}}, \qquad F_{xi,k-1/2} = \frac{\Delta x'_{i,k-1/2}}{\Delta x_{i,k-1/2}},$$

2. Implementation the thin-wall method on z-coordinates

Discretization of 2D flux-form equations with the thin-wall method

$$\left(\frac{\partial \rho'}{\partial t}\right)_{i,k} = -\delta_x\left(F_z \rho u\right)_{i,k} - \delta_z\left(F_x \rho w\right)_{i,k}$$

$$\left(\frac{\partial \rho u}{\partial t}\right)_{i+1/2,k} = -\delta_x\left(F_z \overline{\rho u}^x \overline{u}^x\right)_{i+1/2,k} - \delta_z\left(F_x \overline{\rho u}^z \overline{w}^x\right)_{i+1/2,k} - \delta_x\left(p'\right)_{i+1/2,k}$$

$$\left(\frac{\partial \rho w}{\partial t}\right)_{i,k+1/2} = -\delta_x\left(F_z \overline{\rho w}^x \overline{u}^z\right)_{i,k+1/2} - \delta_z\left(F_x \overline{\rho w}^z \overline{w}^z\right)_{i,k+1/2} - \delta_z\left(p'\right)_{i,k+1/2} - \overline{\rho'}^z_{i,k+1/2} g$$

$$\left(\frac{\partial \rho e^{tot}}{\partial t}\right)_{i,k} = -\delta_x\left(F_z \overline{\rho e^{tot}}^x u\right)_{i,k} - \delta_z\left(F_x \overline{\rho e^{tot}}^z w\right)_{i,k} - \delta_x\left(F_z \overline{p}^x u\right)_{i,k} - \delta_z\left(F_x \overline{p}^z w\right)_{i,k}$$

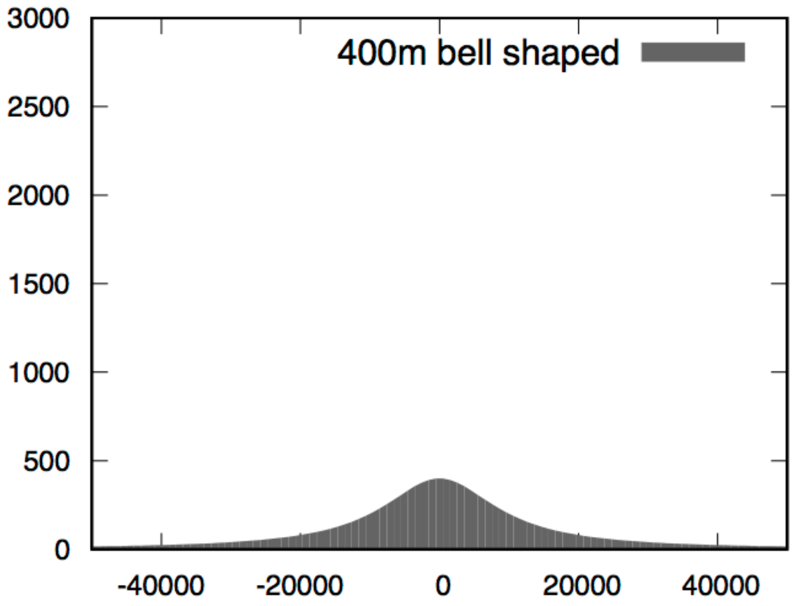
$$\delta_z\left(\phi\right)_{i,k} = \left(\phi_{i,k+1/2} - \phi_{i,k-1/2}\right)/\Delta z_{i,k}, \qquad \delta_z\left(\phi\right)_{i,k} = \left(\phi_{i,k+1/2} - \phi_{i,k-1/2}\right)/\Delta z_{i,k},$$

Nishikawa and Satoh (2016)

2. Isolated mountain wave test

Profile of an isolated mountain

$$h(x) = \frac{h_t a^2}{x^2 + a^2} \qquad \begin{array}{l} a : 10\,km \\ h_t : 400\,m \end{array}$$



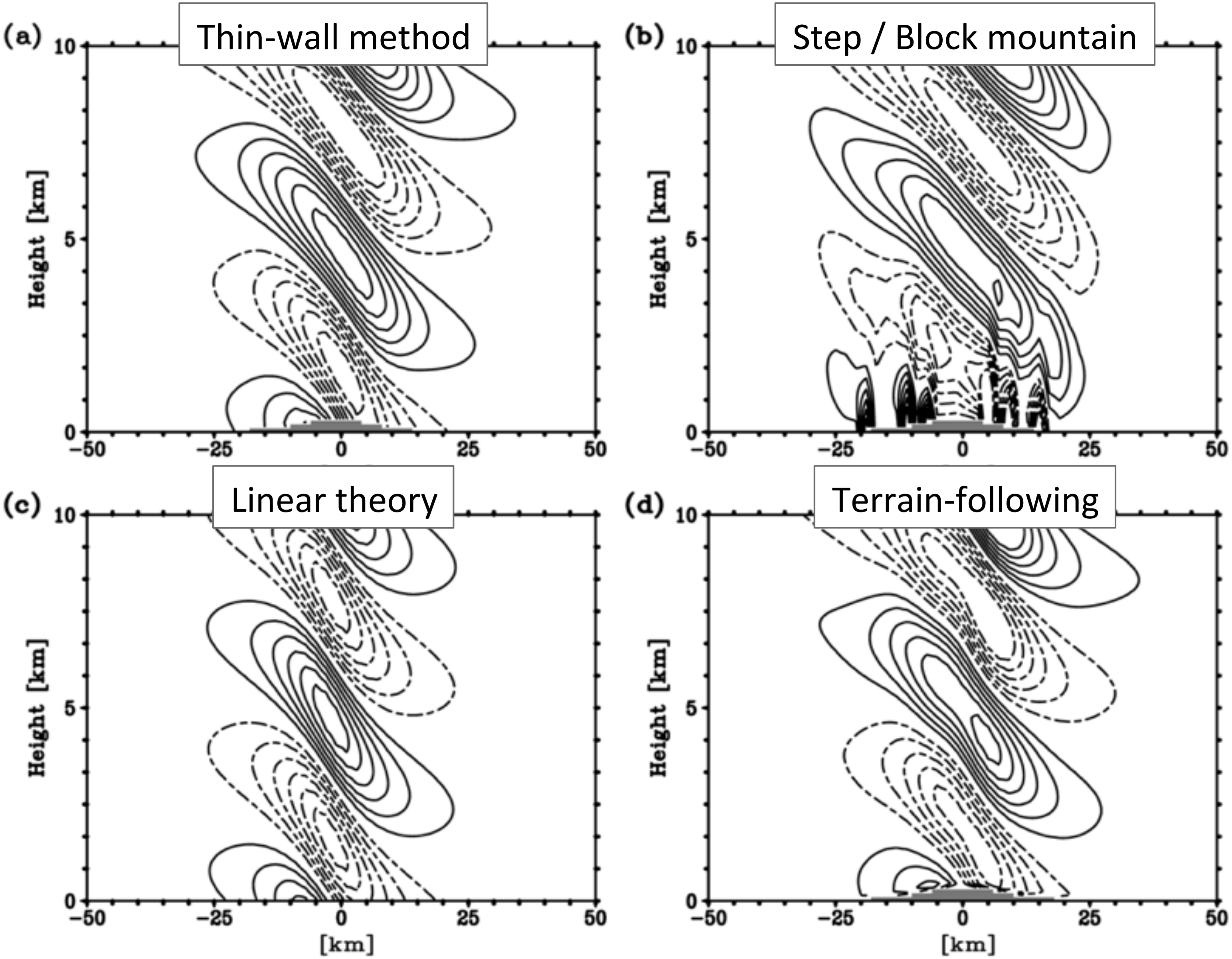
Initial value

- Mean flow U(z) = 10 m/s
- Brunt–Väisälä frequency N = 0.01

Numerical schemes

- Time integration: 3rd order Runge-Kutta scheme
- Spatial difference: 2nd order central difference scheme
- Explicit method in horizontal and vertical
- 4th order numerical diffusion
- Resolution: dx = 2km , dz = 100m

2. Results(w) of an isolated mountain wave test @10H



Thin-wall method

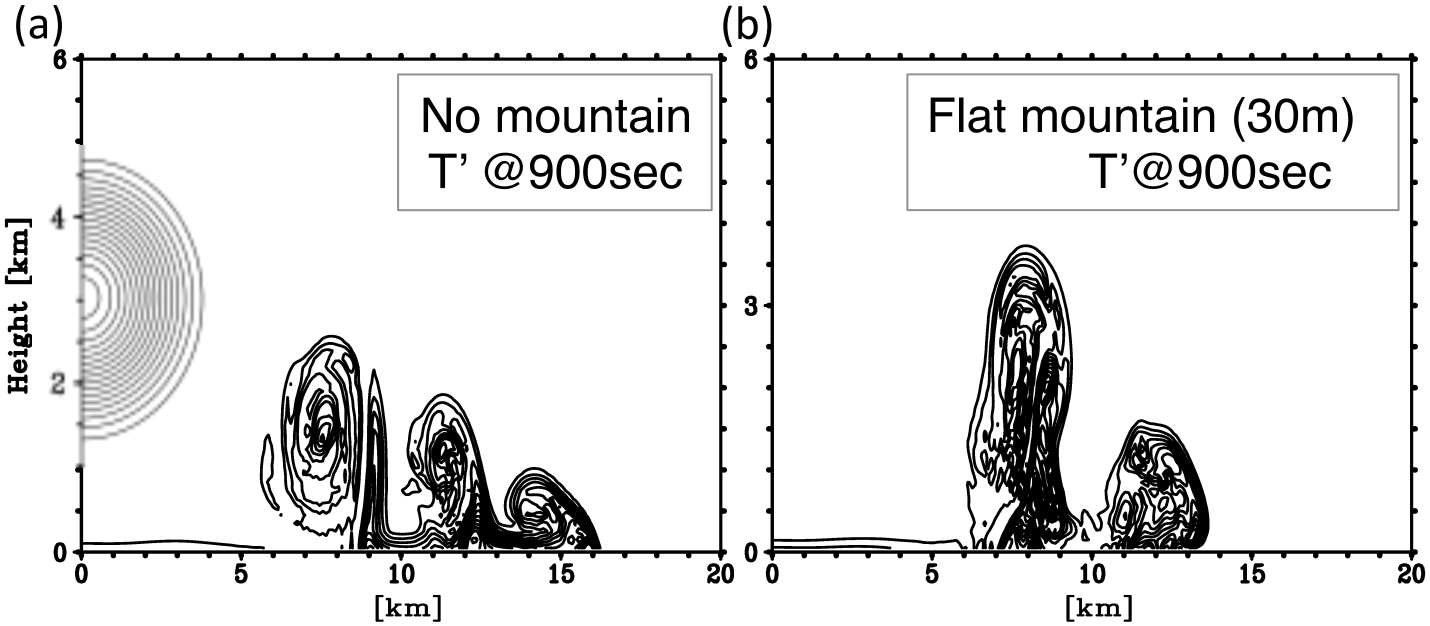
Step / Block mountain

Linear theory

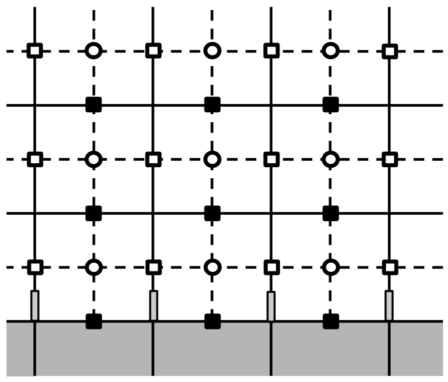
Terrain-following

Contour 0.06 m/s

2. Cold bubble test with the thin-wall method



Thin-wall for scaler



Resolution: Δx = 2km , Δz = 100m
Mountain profile: H(x) = 30 m

Thin walls act as obstruction to the smooth flow.
Instead of going to the horizontal direction, it blows up to the vertical component.

Purpose

- The thin-wall method on z-coordinates performs well in the steady flow test.
- However, it undesirably affects to unsteady flow over a smoother topography (the height is less than Δz).

We will combine the thin-wall method and terrain-following coordinates.

Smoother part: terrain-following coordinates
Steeper part: a thin-wall method

*After here, we call this scheme “the combined method”.

3. How to represent topography by two combined schemes?

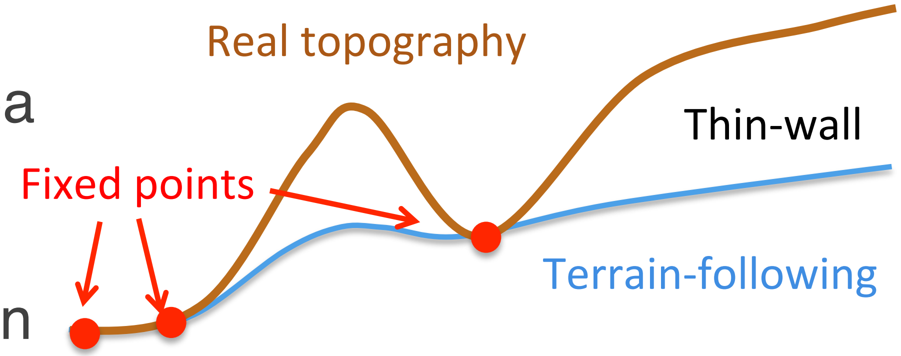
1. Defining local minimum values and land surface in certain area to be fixed.

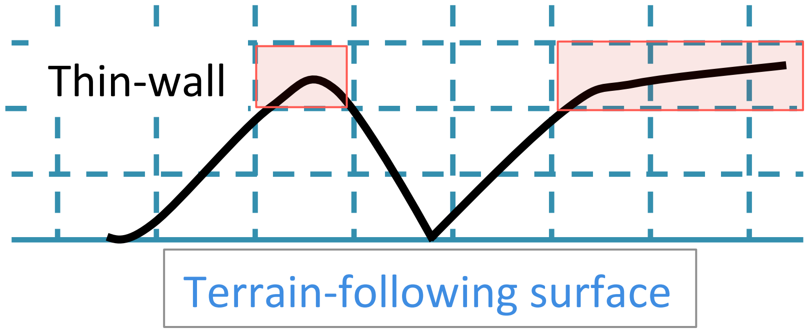
   Exp. valleys, basins

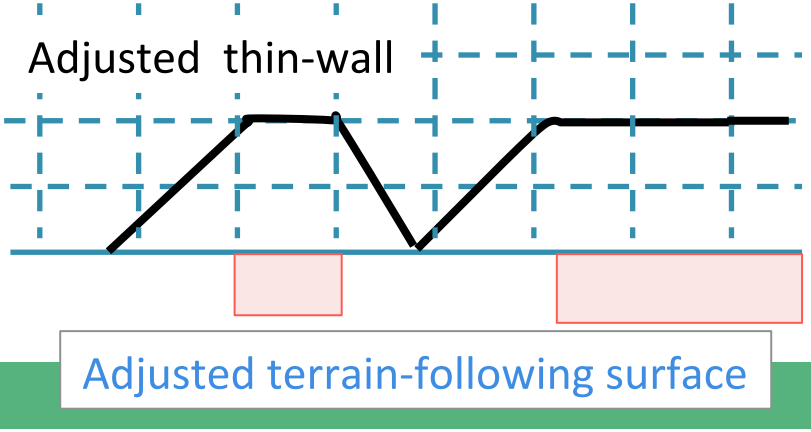
2. Smoothing topography between fixed points until gradient less “MAX GRAD.” .(Numerical filtering for the terrain-following)

3. Cutting off thin walls where a gradient $|\Delta h/\Delta x|$ is less than $\Delta z/\Delta x$.

4. Adding the cut-off parts to the terrain-following coordinates.







3. Thin-wall approximation on terrain-following coordinates

Discretization of 2D flux-form equations with the combined method.

$$\left(\frac{\partial G^{1/2}\rho'}{\partial t}\right)_{i,k} = -\delta_x\left(F_z G^{1/2}\rho u\right)_{i,k} - \delta_z\left(F_z J_{13}\overline{\overline{\rho u}^{x}}^{z} + F_x J_{33}\rho w\right)_{i,k}$$

$$\left(\frac{\partial G^{1/2}\rho u}{\partial t}\right)_{i+1/2,k} = -\delta_x\left(F_z G^{1/2}\overline{\rho u}^{x}\,\overline{u}^{x}\right)_{i+1/2,k} - \delta_z\left(F_z J_{13}\overline{\rho u}^{z}\,\overline{u}^{z} + F_x J_{33}\overline{\rho u}^{z}\,\overline{w}^{x})\right)_{i+1/2,k}$$

$$-\delta_x\left(G^{1/2}p'\right)_{i+1/2,k} - \delta_z\left(J_{13}\overline{p'}^{z}\right)_{i,k+1/2}$$

$$\left(\frac{\partial G^{1/2}\rho w}{\partial t}\right)_{i,k+1/2} = -\delta_x\left(F_z G^{1/2}\overline{\rho w}^{x}\,\overline{u}^{z}\right)_{i,k+1/2} - \delta_z\left(F_z J_{13}\overline{\rho w}^{z}\,\overline{\overline{u}^{x}}^{z} + F_x J_{33}\overline{\rho w}^{z}\,\overline{w}^{z}\right)_{i,k+1/2}$$

$$-\delta_z\left(J_{33}p'\right)_{i,k+1/2} - G^{1/2}\overline{\rho'}^{z}_{i,k+1/2}\,g$$

$$\left(\frac{\partial G^{1/2}\rho e^{tot}}{\partial t}\right)_{i,k} = -\delta_x\left(F_z G^{1/2}\overline{\rho e^{tot}}^{x}u\right)_{i,k} - \delta_z\left(F_z J_{13}\overline{\rho e^{tot}}^{z}\,\overline{\overline{u}^{x}}^{z} + F_x J_{33}\overline{\rho e^{tot}}^{z}w)\right)_{i,k}$$

$$-\delta_x\left(F_z G^{1/2}\overline{p}^{x}u\right)_{i,k} - \delta_z\left(F_z J_{13}\overline{p}^{z}\,\overline{\overline{u}^{x}}^{z} + F_x J_{33}\overline{p}^{z}w)\right)_{i,k}$$

3. Mountain superposed of two different scales.

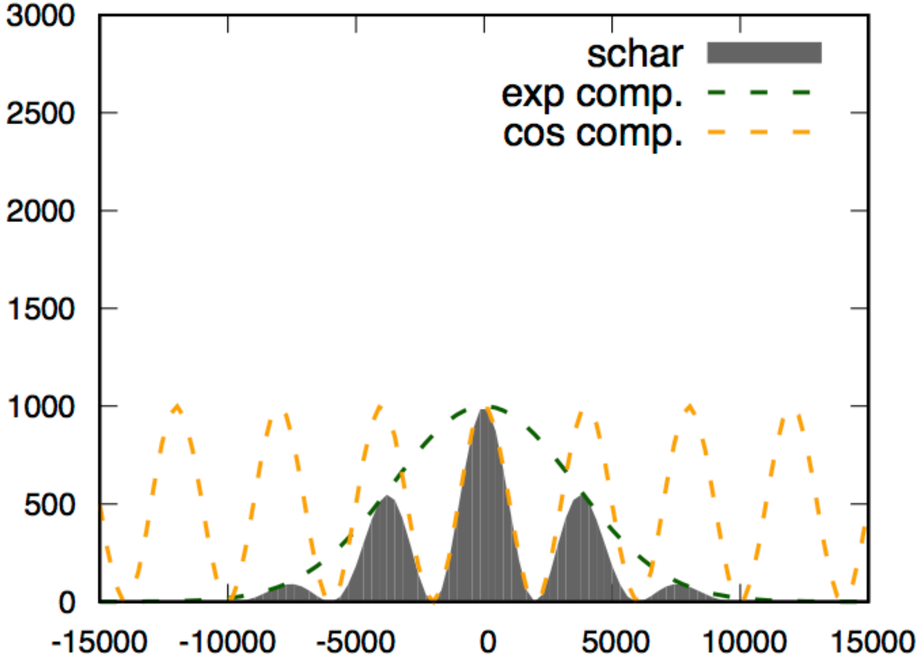
Structure defined by following:

$$h(x) = h_t \exp\left[-\left(\frac{x}{a}\right)^2\right]\cos^2\left(\frac{\pi x}{\lambda}\right)$$

$a : 5\,km$

$h_t : 1000\,m$

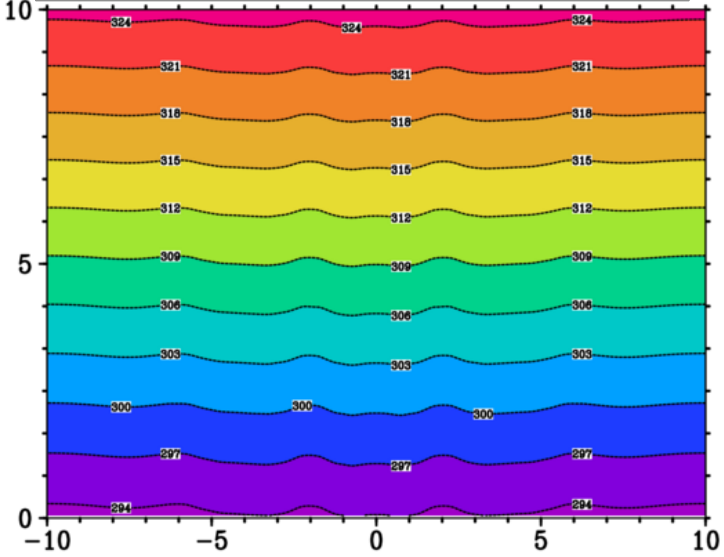
$\lambda : 4\,km$



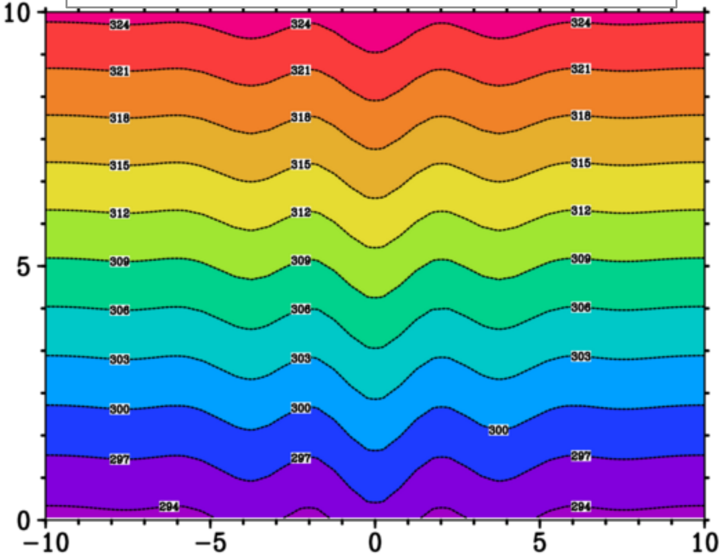
Exp component: a larger-scale hydrostatic wave.
Cos component: a smaller-scale wave characterized by rapid decay with height.

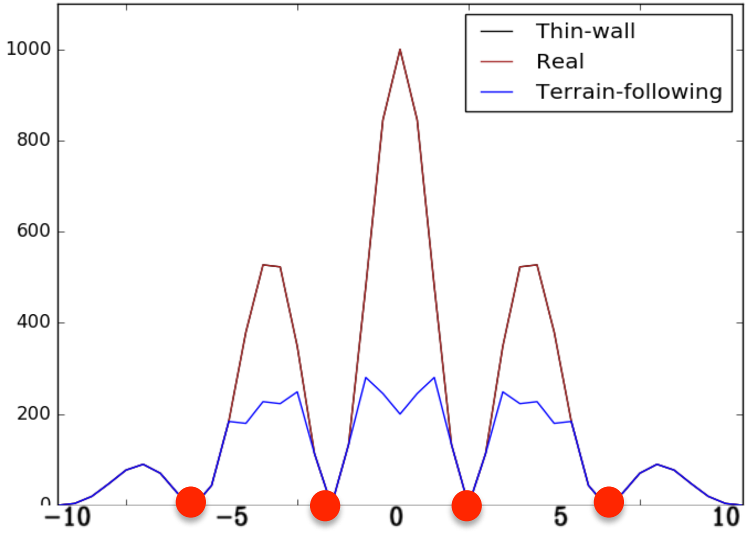
3. Vertical structures & adjusted topography

θ surface: combined method



θ surface: terrain-following

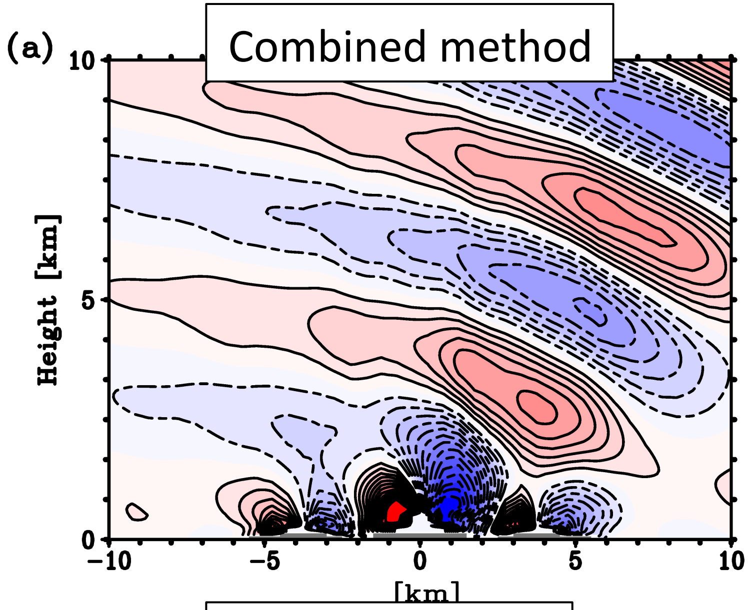


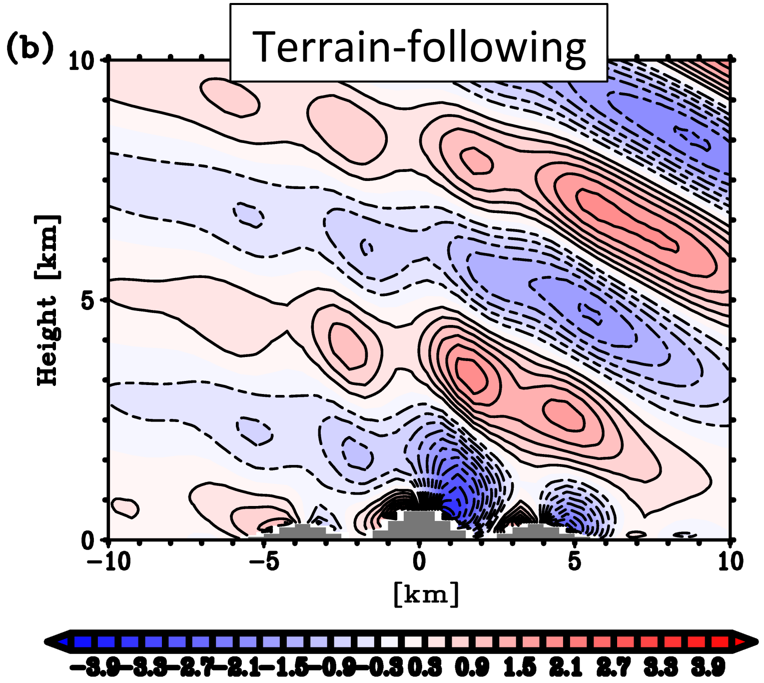


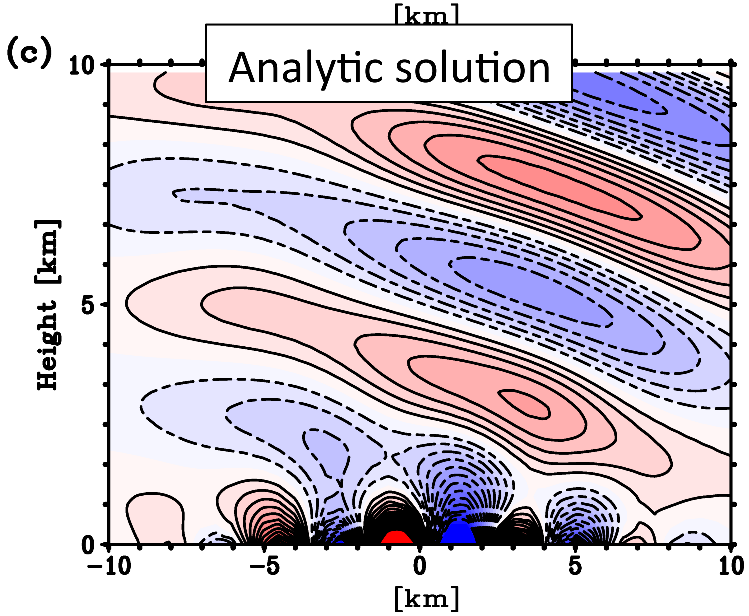
The combined method reduces the distortions of a vertical structure much than terrain-following coordinates.

Max. gradient: 0.01
Resolution: 500m x 100m

Result(w) with the mountain enveloped two components







Contour : 0.3m/s
w@2 hors, resolution 500m x 100m

Combined method : consistent
Terrain-following: spurious mode due to distortion of the vertical structure

Summary

✓ We formulate a combined method, which is combined a thin-wall method and terrain-following coordinate.

✓ We implement a combined method to 2D flux-form equations.

✓ The combined method represents both small- and large-scale gravity waves.

Future works

- Implementing this method to 3D models
- Calculating with moist air