

Examining the sensitivity of the accuracy of EFSO to ensemble size

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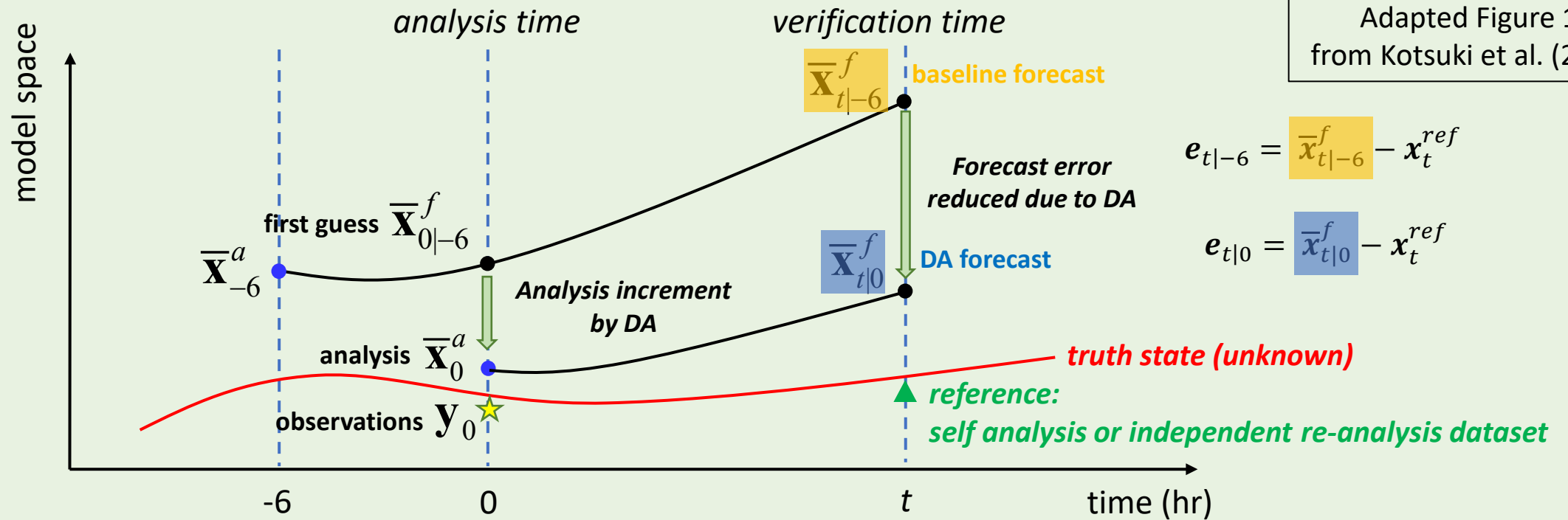
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Background

- EFSO, which stands for **Ensemble Forecast Sensitivity to Observation**, is a method that quantifies the impact of assimilated observations using **ensemble of forecasts**.
- The idea originates from computing the **error reduction between two forecasts: baseline forecast vs. DA forecast**



Forecast error reduction $\Delta e^2 = (e_{t|0}^2 - e_{t|-6}^2) = \mathbf{e}_{t|0}^T \mathbf{C} \mathbf{e}_{t|0} - \mathbf{e}_{t|-6}^T \mathbf{C} \mathbf{e}_{t|-6} = \left(\bar{\mathbf{x}}_{t|0}^f - \bar{\mathbf{x}}_{t|-6}^f \right)^T \mathbf{C} \left(\bar{\mathbf{x}}_{t|0}^f + \bar{\mathbf{x}}_{t|-6}^f - 2\mathbf{x}_t^{ref} \right)$

Background (cont.)

True forecast error reduction (**per grid point j**): $\Delta e_{true,j}^2 = \left(\bar{\mathbf{x}}_{t|0}^f - \bar{\mathbf{x}}_{t|-6}^f \right)_j^T C_{jj} \left(\bar{\mathbf{x}}_{t|0,j}^f + \bar{\mathbf{x}}_{t|-6,j}^f - 2\mathbf{x}_{t,j}^{ref} \right)_j$

Linear error growth assumption
Ensemble error covariance approximation

EFSO estimated forecast error reduction (**per grid point j per observation l**):

$$\Delta e_{EFSO,j,l}^2 = \underbrace{\frac{1}{m-1}}_{\text{ensemble size}} \underbrace{(\delta \mathbf{y}_0)^T}_l \cdot \underbrace{\boldsymbol{\rho}_{l,j}}_{\text{localization matrix}} \left[\underbrace{(\mathbf{R}^{-1} \mathbf{Y}_0^a \mathbf{X}_{t|0}^f)^T}_{\text{Ensemble forecast perturbation analysis perturbation in obs space}} \right]_{l,j} C_{jj} \left(\bar{\mathbf{x}}_{t|0}^f + \bar{\mathbf{x}}_{t|-6}^f - 2\mathbf{x}_t^{ref} \right)_j$$

- Like any ensemble methods, EFSO also suffers from sampling error due to the use of limited-sized ensemble.



- As such, covariance localization is used to suppress sampling error.
- With the Fugaku supercomputing resource, we can afford to run large ensemble ($O(10^3)$) experiments and examine the sensitivity of the accuracy of EFSO to ensemble size and the localization length.

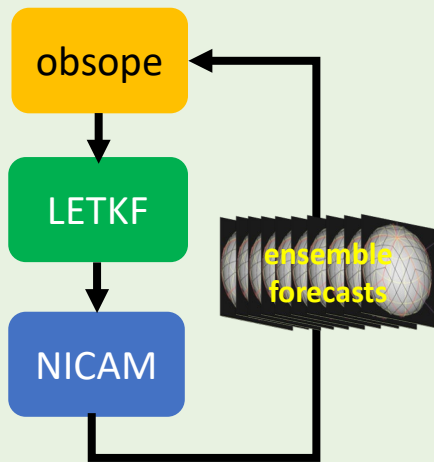
Methodology & Results

NICAM-LETKF settings:

- GL06 ($\Delta x = 112$ km) with 38 vertical levels
- Cycling interval: 6 hour
- Assimilated obs: Conventional, AMSU-A, MHS
- Localization length: H 400 km; V 0.4 ln(p)

$$\Delta e_{EFSO,j,l}^2 = \frac{1}{m-1} (\delta y_0)^T \cdot \rho_{l,j} \left[(R^{-1} Y_0^a X_{t|0}^f)^T \right]_{l,j} C_{jj} (\bar{x}_{t|0}^f + \bar{x}_{t|-6}^f - 2x_t^{ref})_j$$

NICAM-LETKF DA cycle with **1024** ensemble members



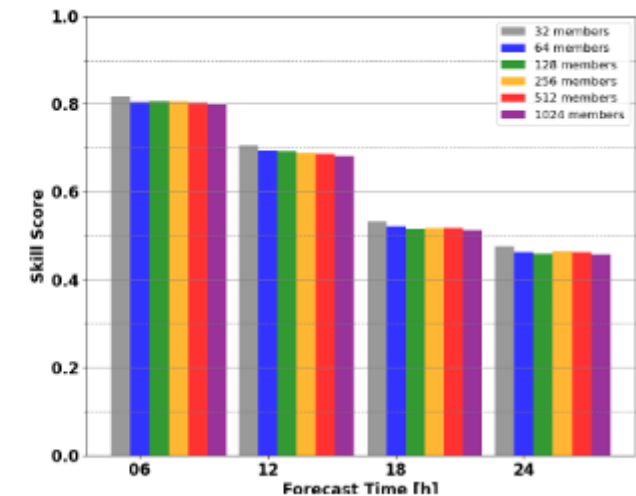
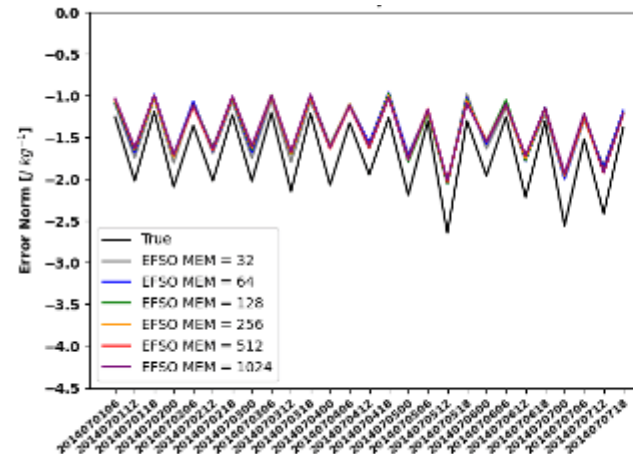
EFSO computation

Compute $Y_0^a X_{t|0}^f$ term using ensemble data sub samples from the 1024 members:

1024, 512, 256, 128, 64, 32 and test different localization lengths $\rho_{l,j}$

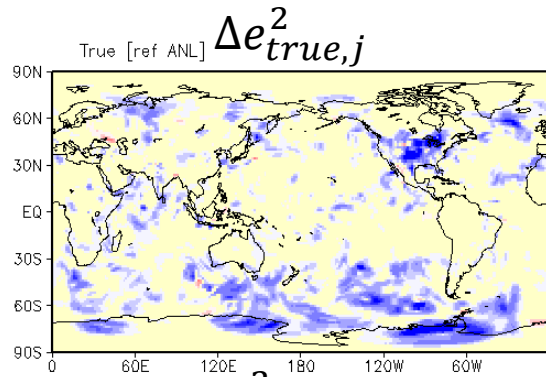
$$\sum_{GL} \Delta e_{true}^2 = \sum_j^{N_{grid}} (\Delta e_{true,j}^2) \text{ v.s. } \sum_{GL} \Delta e_{EFSO}^2 = \sum_j^{N_{grid}} (\Delta e_{EFSO,j}^2)$$

$$skill\ score = 1 - \frac{[\sum_{k=1}^{N_{cycle}} (\sum_{GL} \Delta e_{true,k}^2 - \sum_{GL} \Delta e_{EFSO,k}^2)^2]^{1/2}}{[\sum_{k=1}^{N_{cycle}} (\sum_{GL} \Delta e_{true,k}^2)^2]^{1/2}}$$



- Comparison in the form of difference between globally summed values (used in literature) suggest little sensitivity of EFSO to ensemble size.
- Counter-intuitive results were obtained where smaller ensemble is more skillfull at EFSO estimations.

More Results using new metrics

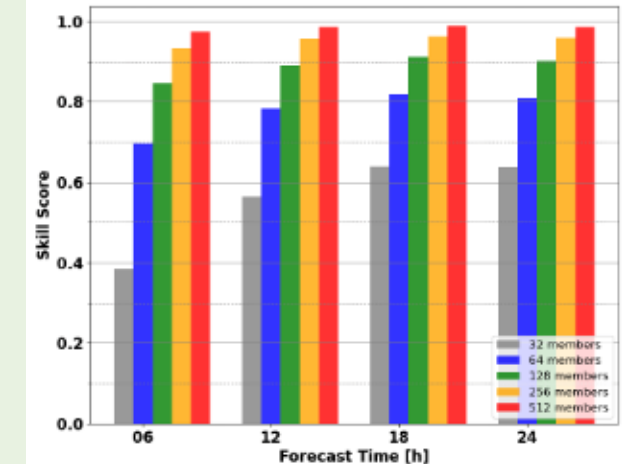
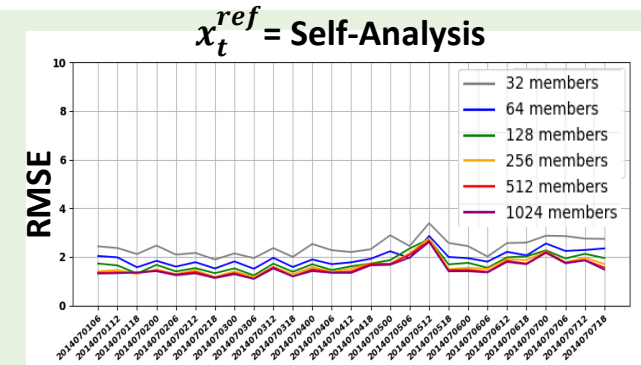
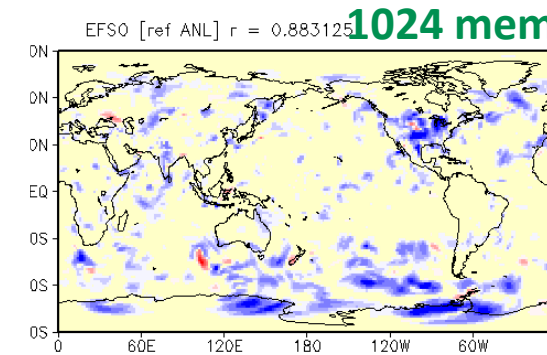
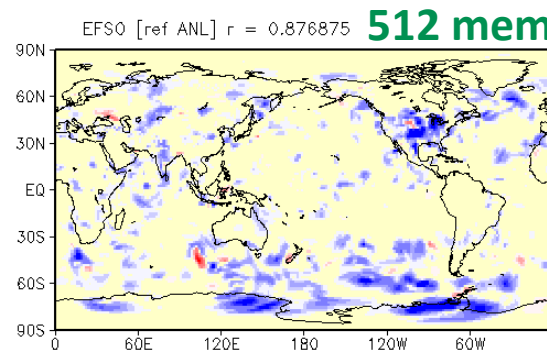
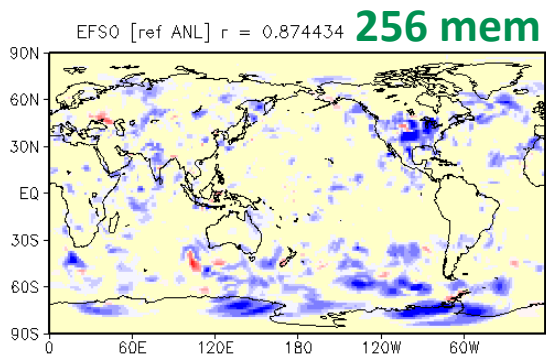
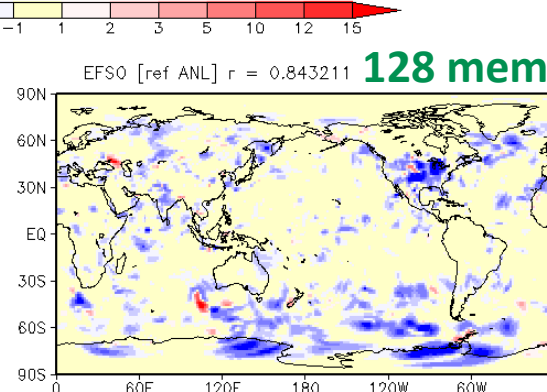
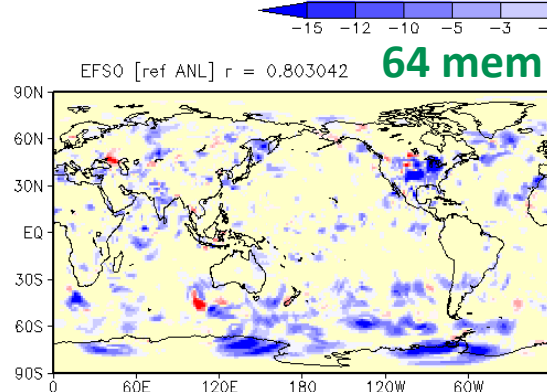
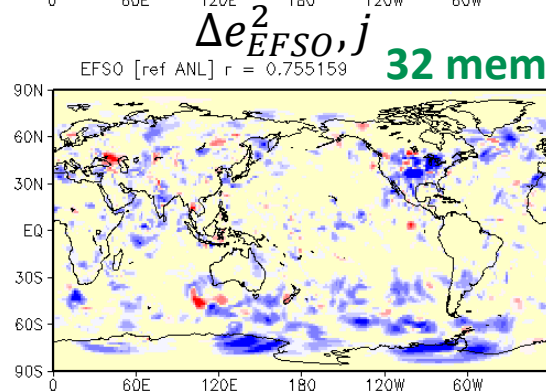


Metrics that account for spatial variation are more relevant:

$$RMSE = \left[\frac{\sum_j^{N_{grid}} (\Delta e_{true,j}^2 - \Delta e_{EFSO,j}^2)^2}{N_{grid}} \right]^{\frac{1}{2}}$$

$$Relative\ Skill\ Score = 1 - \frac{RMSE_m - RMSE_{1024}}{RMSE_{1024}}$$

Corr = Pearson's correlation coefficient between $\Delta e_{true,j}^2$ and $\Delta e_{EFSO,j}^2$



Correlation Coefficient		
0.7552	0.8030	0.8432
0.8744	0.8769	0.8831

More Results on Fraction of Beneficial Observations (FBO)

FBO = # beneficial / (# beneficial + # detrimental)
(this definition excludes neutral obs)

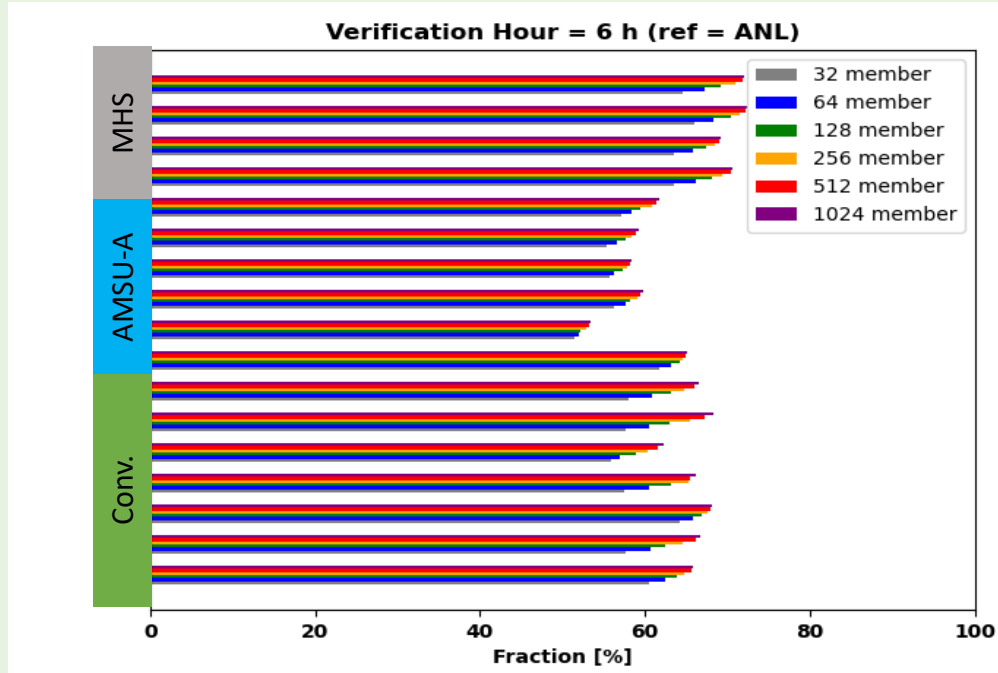
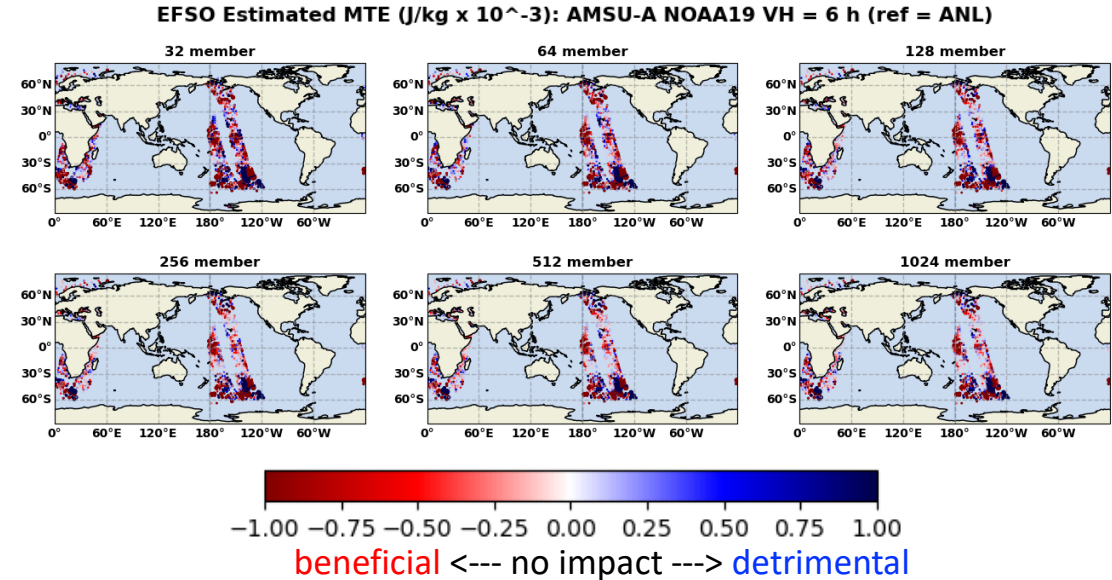


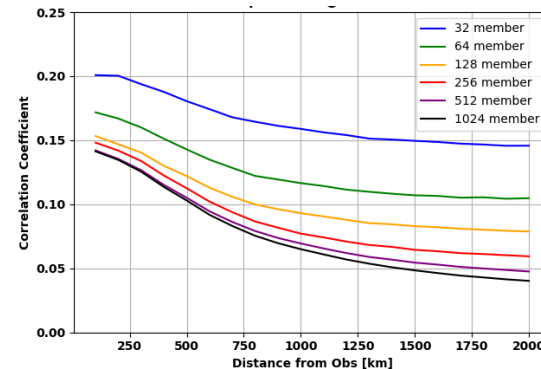
TABLE 3 The time-mean fraction of beneficial observations (%) in CTRL averaged from 4 to 31 July 2014

Kotsuki et al. (2019)	Verification reference				
	FT (h)	CTRL	TWIN-EXP	ERA-Interim	AMSU-A
Fraction of beneficial observations (%)	06	58.9	54.4	55.4	51.9
	12	56.1	54.0	54.3	52.2

➤ Larger ensemble size leads to increased FBO

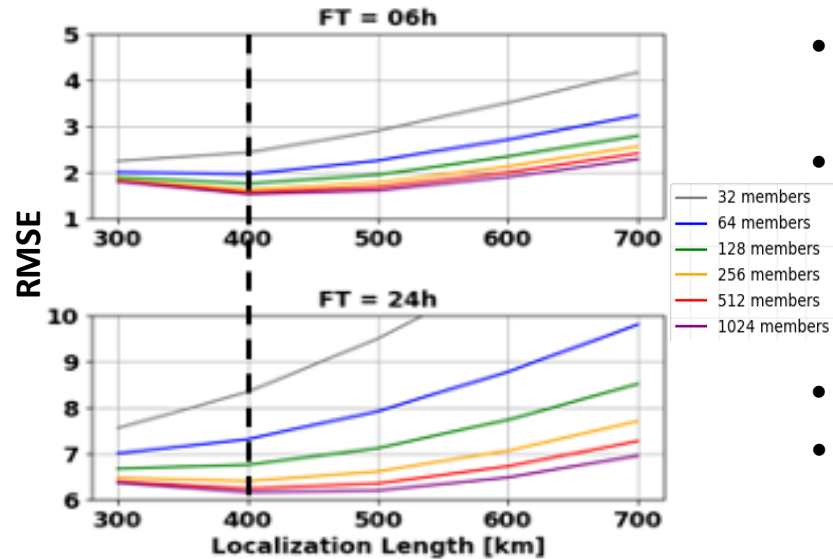


- As # of ensemble members increases, more obs changes from positive (detrimental) to neutral (no impact): Less detrimental observations, larger FBO.

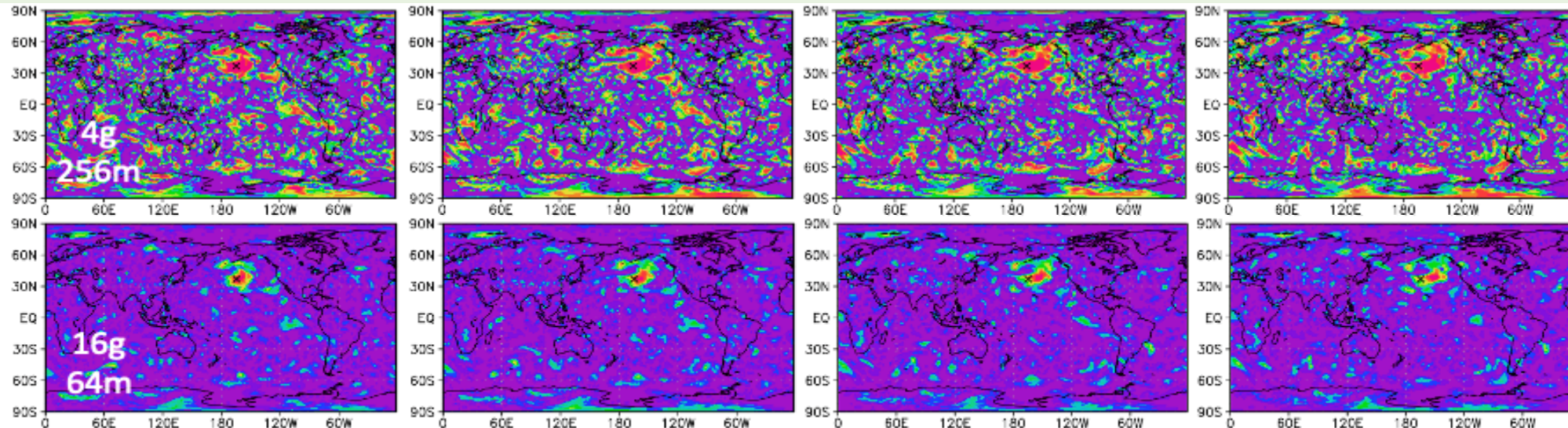


Averaged absolute corr. coef
(normalised $Y_0^a X_{t|0}^f T$)
between AMSU-A NOAA19
and Temperature @ 500 hPa

More Results on Localization & Summary



- When estimating EFSO accuracy in the form of difference between globally summed values, it suggests little sensitivity and leads to counter-intuitive result.
- Considering spatial variation of Δe^2 , three metrics are used: RMSE, Correlation coefficient, and Relative skill score:
 - Confirm sensitivity: **larger ensemble has smaller RMSE and higher correlation**
 - Ensemble of **128 or more can capture >80 % of 1024 ensemble performance**
- We also found that **using larger ensemble size in EFSO leads to larger FBO**
- To address the evolution of localization, **simply increasing localization length does not lead improved EFSO**



Applying a dynamical localization function based on Regression Confidence Factor (RCF) to EFSO is currently under investigation.

Thanks for your attention!