2017 Atmospheric Physics II: Report Problems

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<u>**Problem A**</u> : Discuss how atmospheric circulation changes as global warming due to increase of anthropogenic gas such as CO_2 in the framework of one-dimensional radiative-convective equilibrium following the subsequent questions.

1. Obtain atmospheric structures in radiative equilibrium of a gray atmosphere and find dependencies of the surface temperature T_s and the temperature at the bottom of the atmosphere T_0 on the total optical depth τ_s. Use the following relation between atmospheric optical depth τ and pressure p as

$$\frac{\tau}{\tau_s} = \left(\frac{p}{p_s}\right)^{1+\alpha},$$

   where incoming solar flux $F = 240$ W/m^2, $\alpha = 4$, and $\tau_s = 4$ for the present atmosphere. Search results for the range $\tau_s = 1$-8. Refer to Satoh (2013) for the gray radiation.

2. Draw moist adiabatic profiles for different values of temperatures at the bottom of the atmosphere T_0 in the range $T_0 = 250$-320 K with a 5 K interval. Temperature lapse rate of moist adiabat and saturation water vapor pressure are given by

$$\Gamma_m = \frac{1 + \dfrac{\varepsilon L}{R_d T}\dfrac{p^*(T)}{p}}{1 + \dfrac{\varepsilon^2 L^2}{C_p R_d T^2}\dfrac{p^*(T)}{p}} \frac{g}{C_p},$$

$$p^*(T) = p^*{}_0 \exp\left[\frac{L}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right].$$

   where $\varepsilon = R_d/R_v$.

3. Find solutions in radiative-convective equilibrium with moist adiabatic adjustment and their dependency on the total optical depth τ_s. Assume $T_s = T_0$ and that the radiative cooling in the atmospheric is given by the Newtonian cooling

$$-Q = -\frac{T - T_R}{\tau_R},$$

   where T_R is a solution in the gray radiative equilibrium given by Problem 1, and τ_R is the radiative damping time.

For the energy balance at the surface, the energy inflow from the atmosphere to the surface is assumed to be unchanged as radiative equilibrium. Thus, the sum of the sensible and latent heat flux at the surface is given by $F = \sigma_B \left(T_{Rs}{}^4 - T_0{}^4 \right)$, where T_{Rs} is the surface temperature in the gray radiative equilibrium obtained by Problem 1. The energy balance is given by

$$\int_{p_T}^{p_s} C_p Q \frac{dp}{g} = F,$$

where p_s is pressure at the bottom of the atmosphere and p_T is pressure at the tropopause.

4. Obtain dependencies of the specific humidity at the bottom of the atmosphere q_0, precipitation P, and convective mass flux M in the radiative-convective equilibrium. These are respectively given by $q_0 = rq^*(T_s)$ where r = 80 % is relative humidity at the bottom of the atmosphere, $P = F / L$ where L is latent heat, and

$$M = \frac{P}{q_0}.$$

Problem B：Discuss solutions of the Walker circulation in terms of fractional area of the upward motion region. Refer to Bretherton and Sobel (2002) for notations.

In the non-dimensional range $0 \le x \le 1$ along the equator, the sea surface temperature is given by $T_s = T_{s0} + \Delta SST \cos \pi x$ [K] , and the upward motion is assumed to exist in the range $0 \le x \le f_c$. The water balance equation, the temperature balance equation, and the continuity equation are given by

$$-\frac{\omega M_q}{g} = -P + E + \frac{\Delta p_T}{g} u \frac{dM_q}{dx},$$

$$-\frac{\omega M_s}{g} = -P + R,$$

$$\frac{du}{dx} = \frac{\omega}{\Delta p_T},$$

respectively, where $M_s = M_{s0} + M_{sp} T$、 $M_q = M_{q0} + M_{qp} q$, and M_{s0}= 3017 J/kg, M_{q0}= 2431 J/kg for the radiative-convective equilibrium, and M_{sp} = 0.0435, M_{qp} = 0.0507. The Weak Temperature Gradient assumption is applicable where T is independent of x, and the Strict Quasi-Equilibrium approximation T=q holds in the convective region $0 \le x \le f_c$.

The surface latent heat flux E, latent heat release due to condensation P, and radiative cooling R are given as follows (units are the same as energy flux W/m^2):

$$E = E_0 + a(T_s - T_{s0}) = E_0 + a\Delta SST \cos \pi x,$$
$$P = \begin{cases} P(x); & 0 \le x \le f_c, \\ 0; & f_c < x \le 1, \end{cases}$$
$$R = F - rP,$$

where E_0 =125 W/m^2, $a = c_q \gamma_s C_p$ = 28 W/m^2/K, and r = 0.2.

1. Solve the water balance equation, and find distributions of M_q for some values of ΔSST.
2. Obtain dependencies of f_c as a function of ΔSST.

Problem C : Discuss the Hadley circulation in the axisymmetric framework. Denote φ as latitude, and the sea surface temperature is given by $T_s = T_{s0} - \Delta T_s |\sin\varphi - \sin\varphi_0|^n$ [K]. ITCZ is concentrated near the latitude φ_0, and the downward mass flux M_d [kg/m^2] is constant within the Hadley circulation region except for ITCZ. The meridional flows of the Hadley circulation are confined near z = 0 and H. Use $T_{s0} = 300$ K、$\Delta T_s = 40$ K、H = 15 km. For the other assumptions, refer to Satoh (1994) and Held and Hou (1980).

1. Obtain the latitudinal distribution of the zonal winds near z = H in the Hadley circulation by assuming that the zonal wind is zero at φ_0.
2. Determine the extents of the Hadley circulation in the northern and southern hemispheres for n =2 as functions of φ_0.
3. Solve the angular momentum balance in the Hadley circulation
$$\frac{1}{R\cos^2\varphi}\frac{\partial}{\partial\varphi}\left(\frac{1}{H}\int_0^H uv\cos^2\varphi dz\right) = -\frac{Cu(0)}{H}$$
to obtain the latitudinal distribution of the zonal winds near z = 0, $u(0)$, by estimating a typical value of C.
4. Discuss characteristics of the Hadley circulations for n = 3 and 4.

Problem D : Using a time-dependent 2-column model following Section 5b of Satoh and Hayashi (1992), obtain vertical profiles of time-averaged convective mass flux $Mc(z)$ by specifying the radiative cooling profile $Q^{rad}(z)$. Assume that the upward motion region is moist adiabatic and is saturated from the surface to the neutral level at z_N. The surface temperature is T_s = 300 K, and the temperature at the bottom of the upward motion region is equal to T_s. The neutral level z_N is the first height where density of the upward motion region becomes the same as that of the downward motion region:

$$\rho^u(z_N) = \rho^d(z_N)、\ \rho^u(z) < \rho^d(z) \quad (0 < z < z_N).$$

The equations for specific humidity and temperature in the downward motion region are given as follows. For $z > z_N$,

$$\rho^d \frac{\partial q^d}{\partial t} = 0,$$

$$C_p \rho^d \frac{\partial T^d}{\partial t} = -Q^{rad},$$

and for $z < z_N$,

$$\rho^d \frac{\partial q^d}{\partial t} = M_c \frac{\partial q_d}{\partial z},$$

$$C_p \rho^d \frac{\partial T^d}{\partial t} = C_p M_c \left(\frac{\partial T^d}{\partial z} + \frac{g}{C_p} \right) - Q^{rad},$$

and within a thin layer near the neutral level $|z - z_N| < \Delta z$,

$$\rho^d \Delta z \frac{\partial q^d}{\partial t} = M_c \left(q^u - q^d \right),$$

$$C_p \rho^d \Delta z \frac{\partial T^d}{\partial t} = C_p M_c \left(T^u - T^d \right) - Q^{rad},$$

where water vaper and energy are transported from the upward motion region to the downward motion region. Convective mass flux is given by

$$M_c = C\sqrt{W},$$

$$W = \int_0^{z_N} \frac{\rho^d - \rho^u}{\rho^d} g dz.$$

For simplicity, $Q^{rad}(z)$ is time independent and constant irrespective of height $0 < z < z_T$ (z_T = 15 km) such that $Q^{rad}(z)$ = 2 K/day. Discretize in the vertical with the vertical resolution about Δz = 100 m, and integrate in time. Determine the solutions for $C = 10^{-3}, 10^{-4}, 10^{-5}$ kg/m^3.

References :

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